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INVESTIGATION OF DIGITAL FILTERING TECHNIQUES  
FOR THE ANALYSIS OF EXPERIMENTAL DATA



BY

LANNY D. WELLS

JUNE 1975

FINAL REPORT



**RESEARCH DIRECTORATE**

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<p>This paper presents techniques of processing digital data to remove unwanted frequencies and to simulate the effect of simple RC networks on the corresponding analog data. The development of these techniques provides: a means of isolating a signal from background noise, a means of removing low frequency drift from a signal, and a means of selectively reversing previous filtering. These techniques are valuable in the refinement and validation of digital simulations. The method presented in this paper uses the Fast Fourier Transform. The methodology has been applied to remove noise and drift from experimental and mathematical</p>		

20.

model data relative to determining an RMS value of a tank gun pointing error.

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## 1.0 INTRODUCTION

The subject of digital filters is somewhat confused because a digital filter is usually considered to be a computational procedure rather than a physically recognizable piece of hardware. These computational procedures vary greatly in complexity and purpose. Some may digitally simulate a real electrical or mechanical filter. Others may simulate idealized filters which could not be easily made into hardware, and still others may be simple numerical smoothing techniques not related to any actual or ideal filter.

The methods presented in this paper are based on the use of the reversible time to frequency transformation - the 'Fast Fourier Transform' (FFT). To put this method into proper perspective, some traditional methods are briefly discussed in the next section.





## 2.0 TRADITIONAL METHODS

Various numerical procedures for smoothing discrete data traces have been called digital filters. Some of these methods simply replace each data point with a weighted average of the surrounding points. The computations are usually done on a digital computer. Another method treats filtering as an initial value problem. That is, a digital simulation of a real mechanical or electrical system is used as a filter. The discrete data record to be filtered is considered to be a forcing function or applied voltage, and a computed force or voltage at some point in the system is the filtered data. A digital computer numerically integrates the system differential equations to obtain the output.

These methods have several disadvantages. The actual effect upon the frequency content of the data being filtered is not intuitively obvious. The effective transfer function of the simple averaging process depends upon the sample spacing as well as the particular set of weighting factors used. In the numerical integration method, the parameters of the system must be carefully chosen to obtain the desired frequency response and scale factors. Both methods may produce some undesirable amplification and/or shifts in the frequencies in the pass band, and the desired transfer function can usually only be approximated. Also, there may be several pass bands because of the phenomenon of frequency aliasing. To avoid these limitations, the following FFT method is becoming popular.



### 3.0 THE FFT METHOD

The Discrete Fourier Transform (DFT) is the basic tool which facilitates the methodology presented in this paper. The DFT is defined by:

$$A_r = \sum_{k=0}^{N-1} X_k \text{ EXP } (-2\pi jrk/N) \quad (1)$$

for  $r = 0, \dots, N - 1$

The usefulness of the DFT was limited because the computational time was too great, even on the faster digital computers. A breakthrough occurred in the mid 60's with the development of a much faster algorithm for computing the DFT. This method is commonly called the 'Fast Fourier Transform' (FFT).<sup>\*</sup> The FFT provides an efficient procedure for transforming a discrete time series into a complex function of frequency. No information is lost in this transformation; in fact, the transformation is reversible. This reversibility feature of the FFT suggests the possibility of modifying a time series by operating on its FFT followed by an inverse FFT. Filtering may be accomplished by the convolution of the FFT and the frequency transfer function of a filter followed by an inverse FFT. This is equivalent to solving the convolution integral to obtain the filter output:

$$C(t) = \int_{-\infty}^t X(\tau) h(t-\tau) d\tau \quad (2)$$

where:

$C(t)$  is the output

$X(t)$  is the input

$h(t)$  is the response of the filter to a step input

The direct solution of equation (1) is completely impractical if it must be done by numerical integration. Even if the input function  $x(t)$  is zero for  $t < 0$  so that the lower limit on the integral becomes zero, the computation time is much too great.

\* The FFT in this study was computed using subroutine FOUR1, written by Norman Brenner of the MIT Lincon Laboratory, July 1967. A listing of FOUR1 is included in the appendix.



#### 4.0 IDEALIZED FILTERS

Ideal or perfect filtering may be accomplished by setting the FFT coefficients corresponding to frequencies outside the pass band to zero followed by an inverse FFT. The passed frequencies are essentially unchanged in both magnitude and phase. Subroutines for this type of filtering (PLPASS, PHPASS, and PBPASS) which are Perfect Low PASS, Perfect High PASS, and Perfect Band PASS filters respectively have been written and are included in the appendix.

Examples of using these filters follow. Figure 1 shows the test data used to demonstrate each of the filters presented in this paper. Figure 2 shows the power spectrum of the test data which consisted of discrete frequencies at 1.0, 2.25, 5.0, 10.0, 20.0, and 30.0 HZ. This hypothetical data was used because it does contain a wide range of strong frequencies, thus, providing the opportunity of demonstrating low-pass, high-pass, and band-pass filters all using the same input data. Figure 3 shows the output from the Perfect Low PASS (PLPASS) filter. The cutoff of 3.625 HZ only allows the frequencies at 1.0 and 2.25 to pass. Figure 4 shows the output from the Perfect High-Pass (PHPASS) filter with a cutoff frequency at 15.0 HZ allowing only the frequencies at 20.0 and 30.0 HZ through. Figure 5 shows the use of the Perfect Band PASS (PBPASS) filter. Note that the low cutoff of 7.5 and the high cutoff of 25.0 HZ allows only the frequencies at 10.0 and 20.0 HZ through.

The selection of cutoff points for filtering real data should be based on some knowledge of the system from which the data was obtained. For example the hull roll motion of a tank has a natural frequency of about 1.0 HZ, thus, any frequency content greater than 5.0 to 10.0 HZ is probably noise and may be filtered out. One source of noise in tank data which has been observed, is caused by the track shoes contacting the ground. This shows up on a power spectrum plot as a sharp peak at 15.0 to 20.0 HZ.

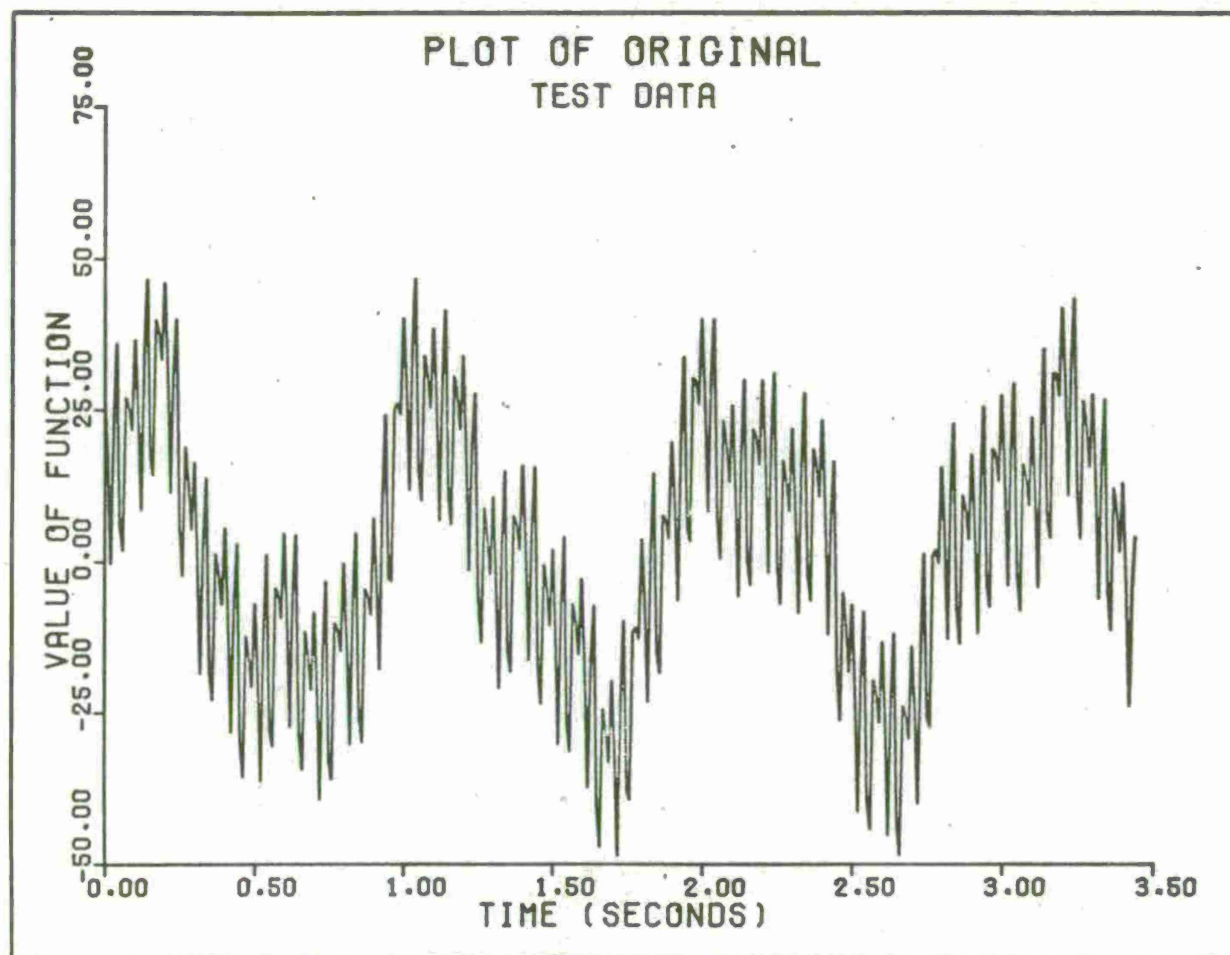


Figure 1 Plot of Original Test Data

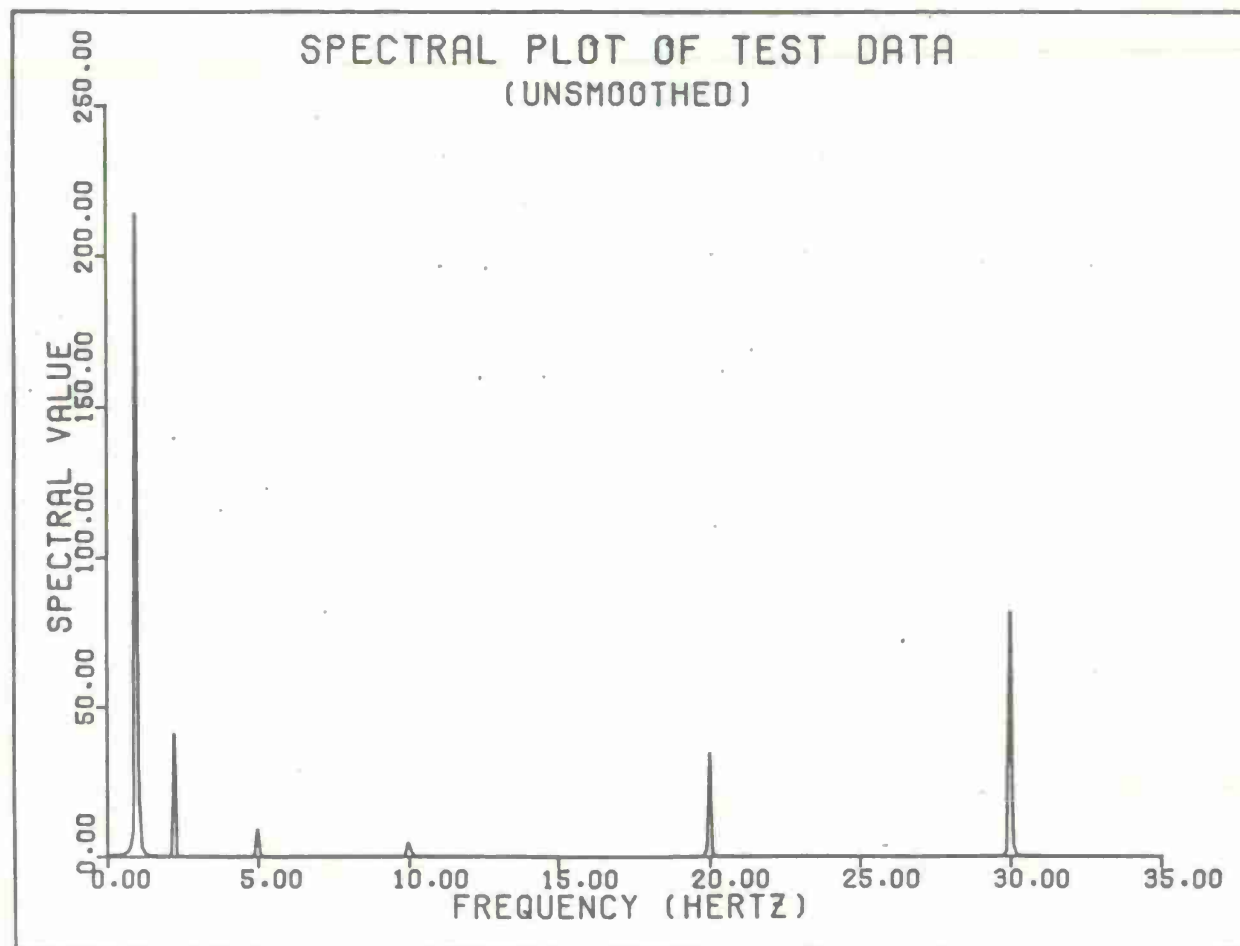


Figure 2 Spectral Plot of Test Data (Unsmoothed)



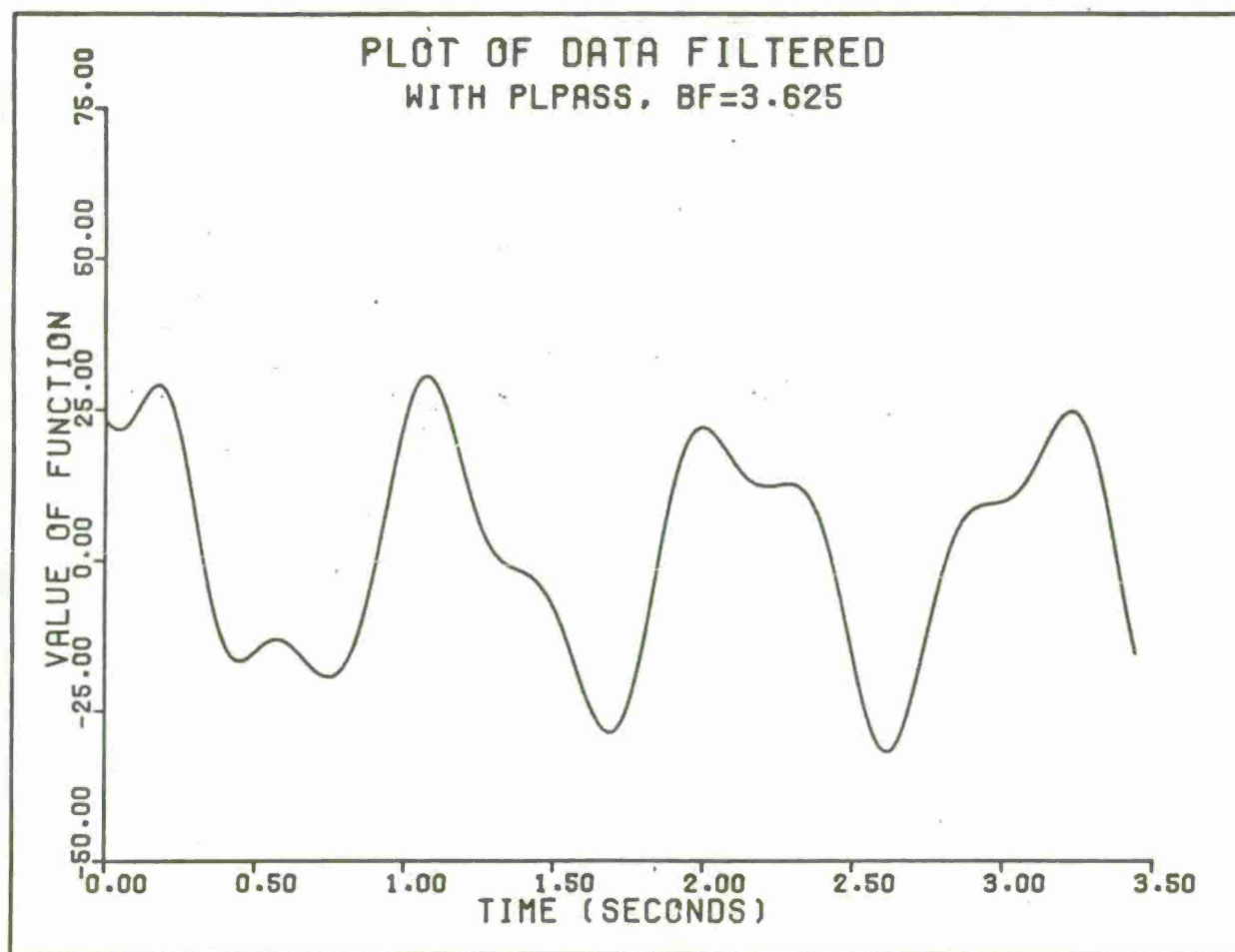


Figure 3 Plot of Data Filtered with PLPASS, BF=3.625

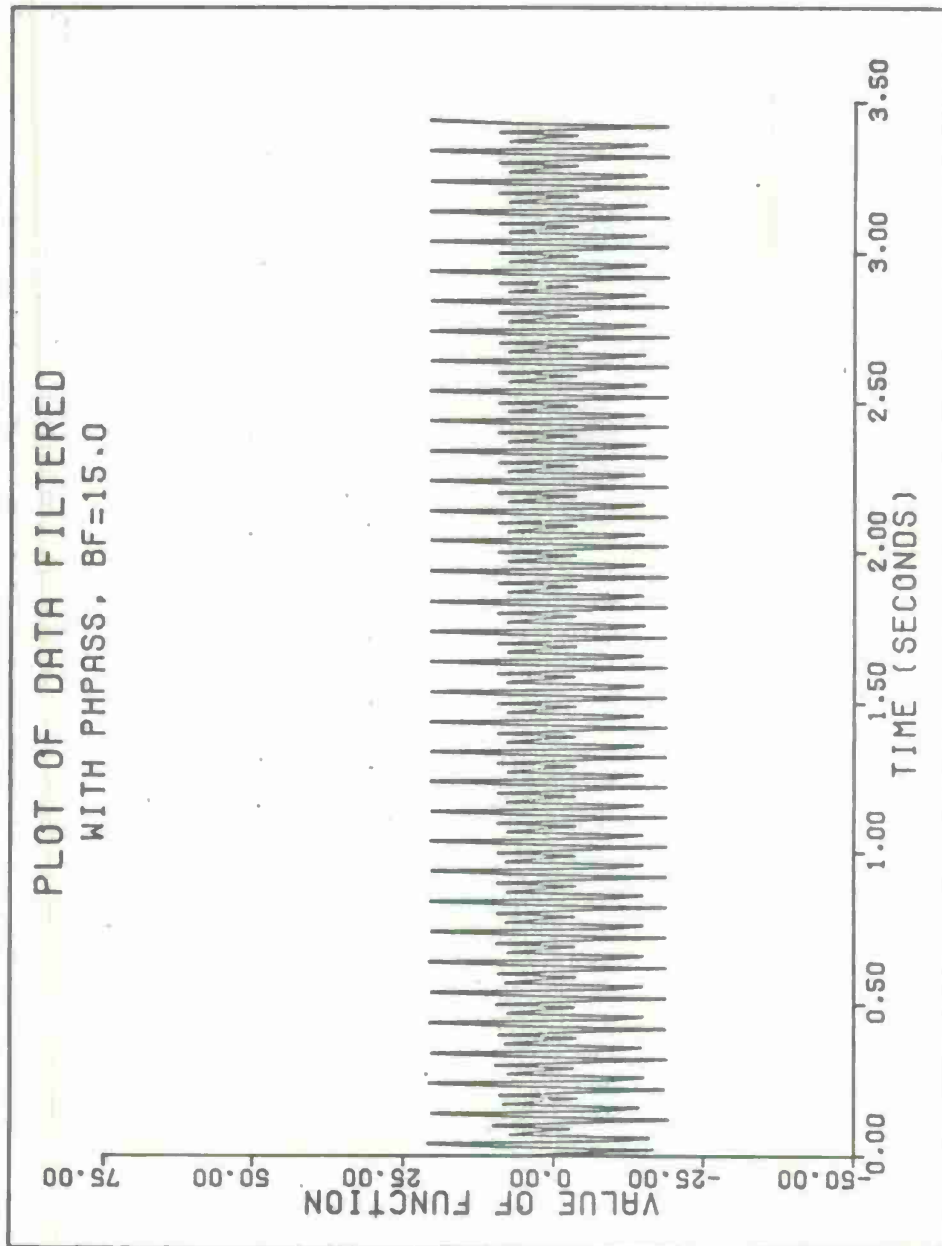


Figure 4 Plot of Data Filtered with PHPASS, BF=15.0

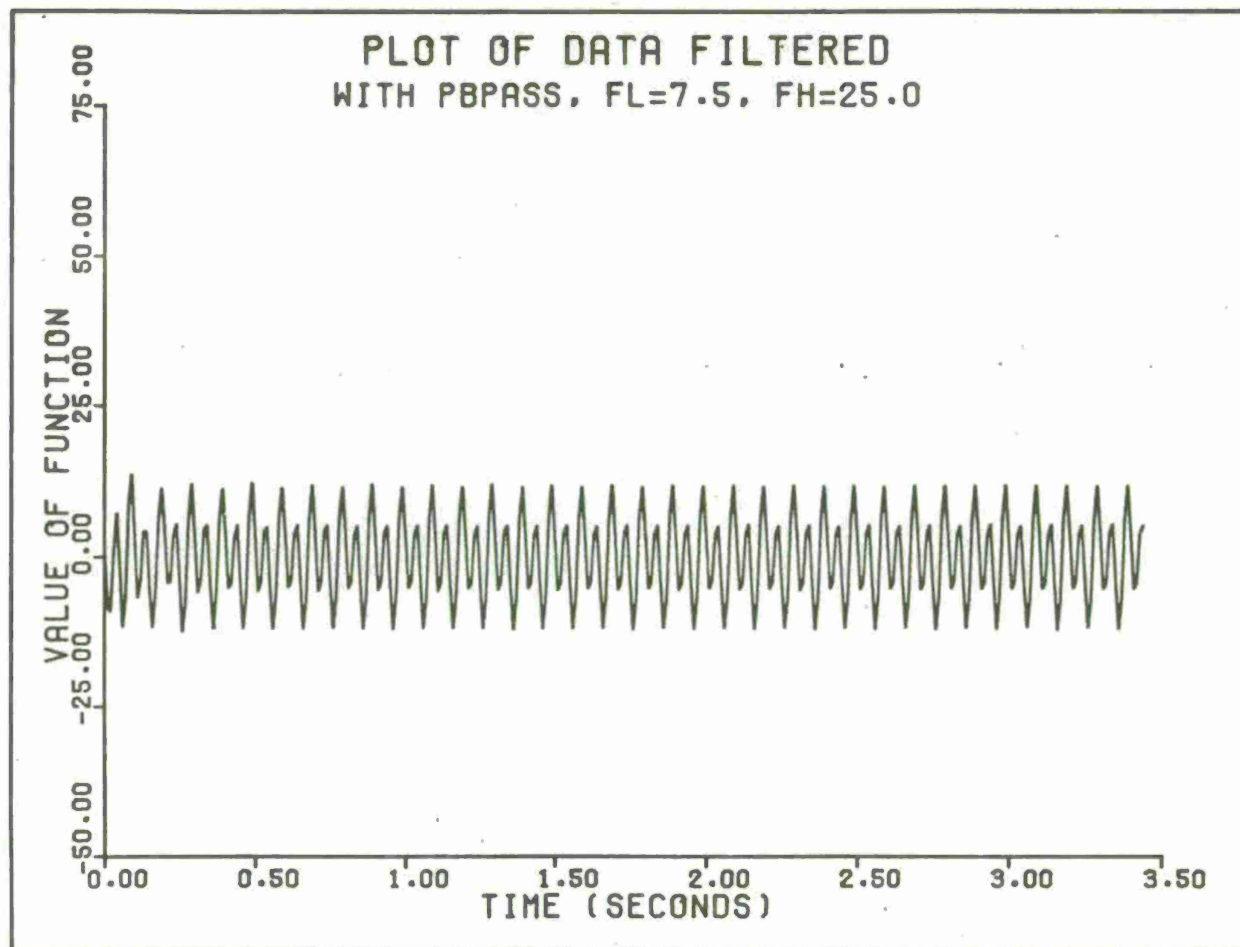
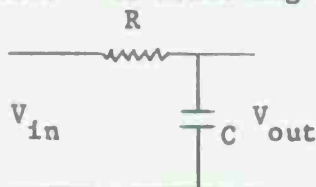


Figure 5 Plot of Data Filtered with PBPASS, FL=7.5, FH=25.0

## 5.0 DIGITAL SIMULATION OF REAL FILTERS

Although the perfect filters are very effective in removing unwanted frequency content of a signal, it may be desirable to only attenuate certain frequency bands rather than suppress them completely. This will retain more of the features of the original signal. This is precisely what a real RC filter does. The following is a simple RC filter:



Neglecting loading effects, the transfer function can be easily obtained:

$$F(j\omega) = \frac{1}{1 + j\omega/BF} \quad (3)$$

where  $\omega$  = frequency

$$j = \sqrt{-1}$$

$$BF = 1/RC$$

Plots of this transfer function for two break frequencies are included in the appendix. To simulate this filter, the FFT of the input is multiplied, point by point, by the transfer function of the filter and the inverse FFT is obtained. A subroutine for this filter (RCN1) has been written and is included in the appendix. Figure 6 shows the output of this filter for a particular break frequency (BF) using the same input data as before. Note that the high frequencies are still present, although attenuated. This fact suggests the possibility of reversing the process. This may be accomplished by the same procedure except with the inverse transfer function:

$$F^{-1}(j\omega) = \frac{v_{in}}{v_{out}} = 1 + j\omega/BF \quad (4)$$

The computations are basically the same, thus, subroutine RCN1 has been written so that it can also reverse the process. This has been done using the data in Figure 6; the result shown in Figure 7 is not distinguishable from the original plot. This feature may be used to recover data which was filtered at too low a frequency during recording or digitizing. Although the original data could be recovered by electrical processing of the original analog data and redigitizing, the analog tape and/or the special equipment and expertise to do this may not be readily available.

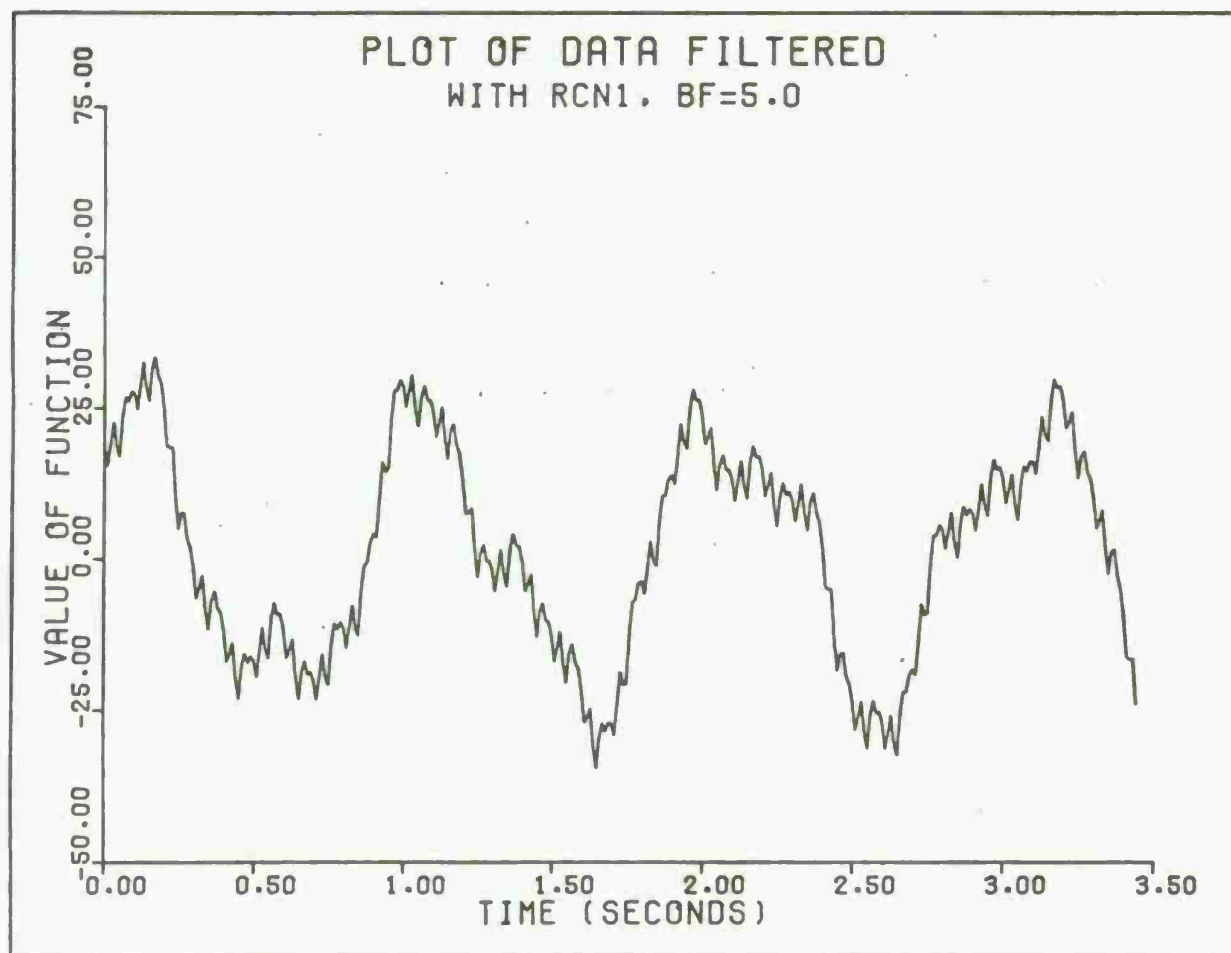


Figure 6 Plot of Data Filtered with RCN1, BF=5.0

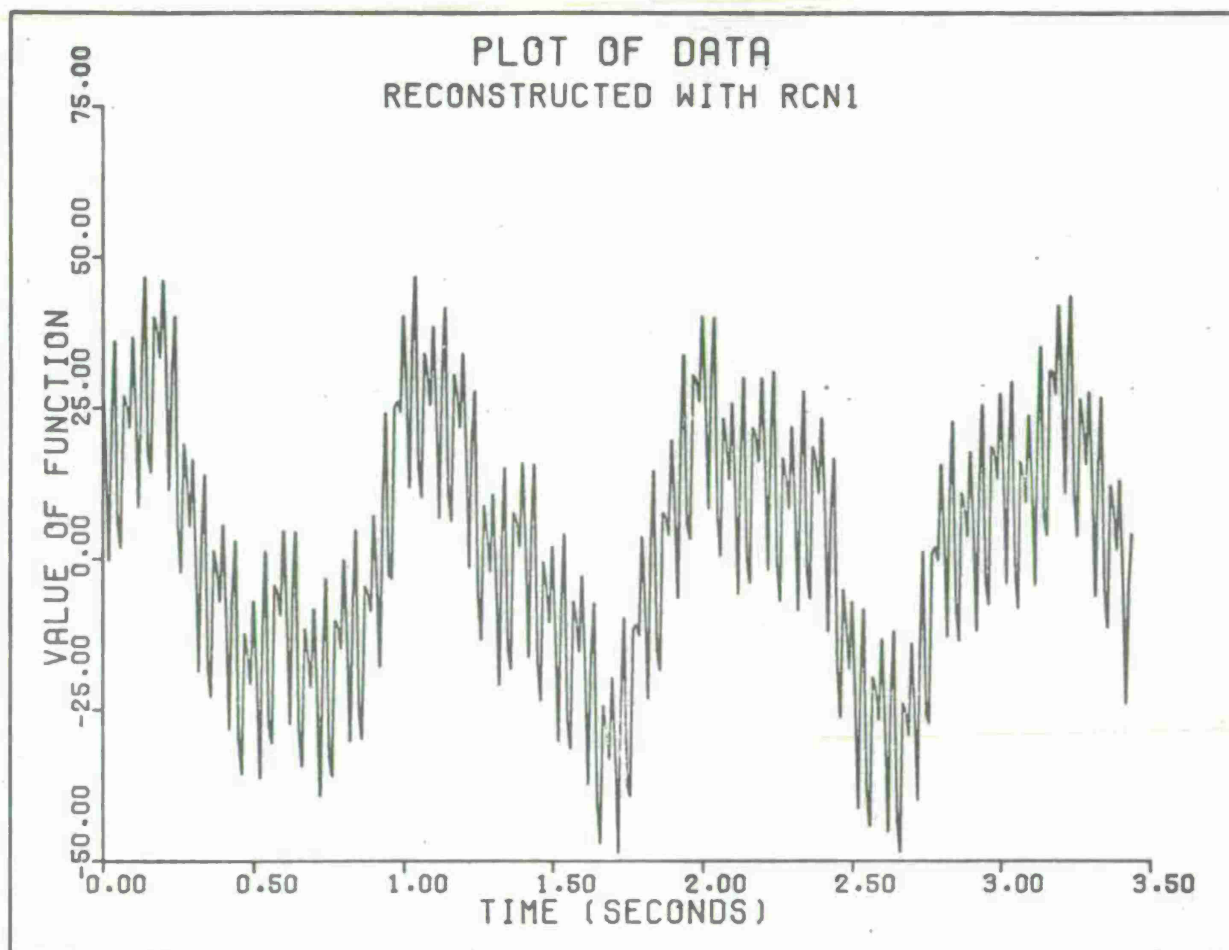
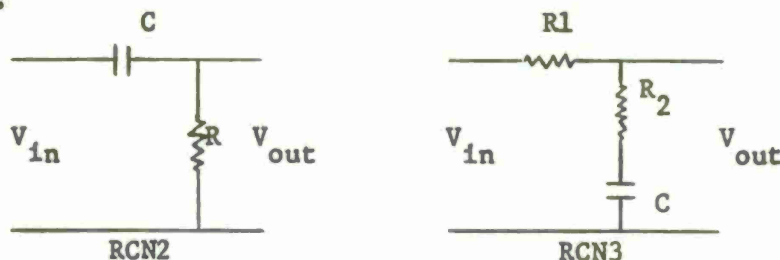


Figure 7 Plot of Data Reconstructed with RCN1

Thus, this digital approach will usually be more expeditious. One potential problem in trying to reconstruct the original signal in this manner is that any noise in the signal is also amplified. To help avoid this problem RCN1 has an optional sharp cut off frequency, CF. Any frequency components greater than CF are set equal to zero as was done in the perfect filters discussed before. If CF is equal to or greater than the Nyquist frequency ( $1.0/2 * DT$ ), which is the highest frequency that can be considered, none of the frequencies are cut off.

Additional subroutines have been written (RCN2 and RCN3) and are included in the appendix. These subroutines correspond to the following RC networks:



RCN2 has the following transfer function:

$$F(j\omega) = \frac{j\omega}{BF + j\omega} \quad (5)$$

where:

$\omega$  = frequency

$BF = 1/RC$

$j = \sqrt{-1}$

This is basically a high-pass filter; it, obviously, will not pass DC. Plots of the transfer function for two values of BF are included in the appendix. Figure 8 shows the result of using RCN2 on the test data plotted in Figure 1.

RCN3 has the following transfer function:

$$F(j\omega) = \frac{1}{A} \frac{BF + j\omega}{BF/A + j\omega} \quad (6)$$

where:

$A = 1 + R1/R2$

$BF = 1/(R2 * C)$

$j = \sqrt{-1}$

$\omega$  = Frequency



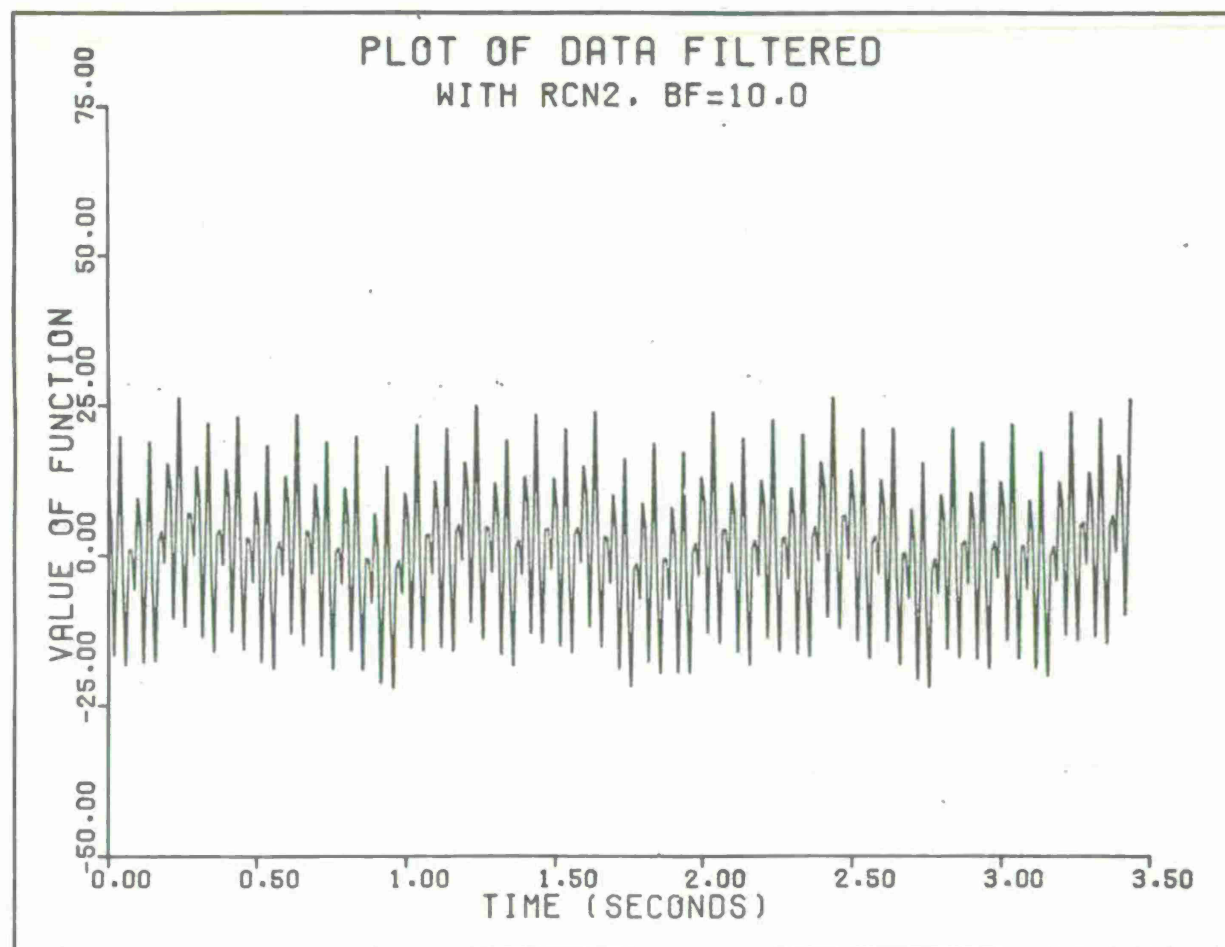


Figure 8 Plot of Data Filtered with RCN2, BF=10.0

This filter is a low-pass filter, similar to RCN1; however, the attenuation curve in the low frequency range is much steeper and the attenuation of high frequencies approaches a fixed value of  $1/A$ . A plot of the transfer function, for particular values of BF and A, is included in the appendix. The use of RCN3 on the test data of Figure 1 is shown in Figure 9. The outputs of both RCN2 and RCN3 were successfully used to reconstruct the original data by using RCN2 and RCN3 with their inverse transfer functions. The resulting plots were indistinguishable from Figure 1 and are shown in Figures 10 and 11 respectively.

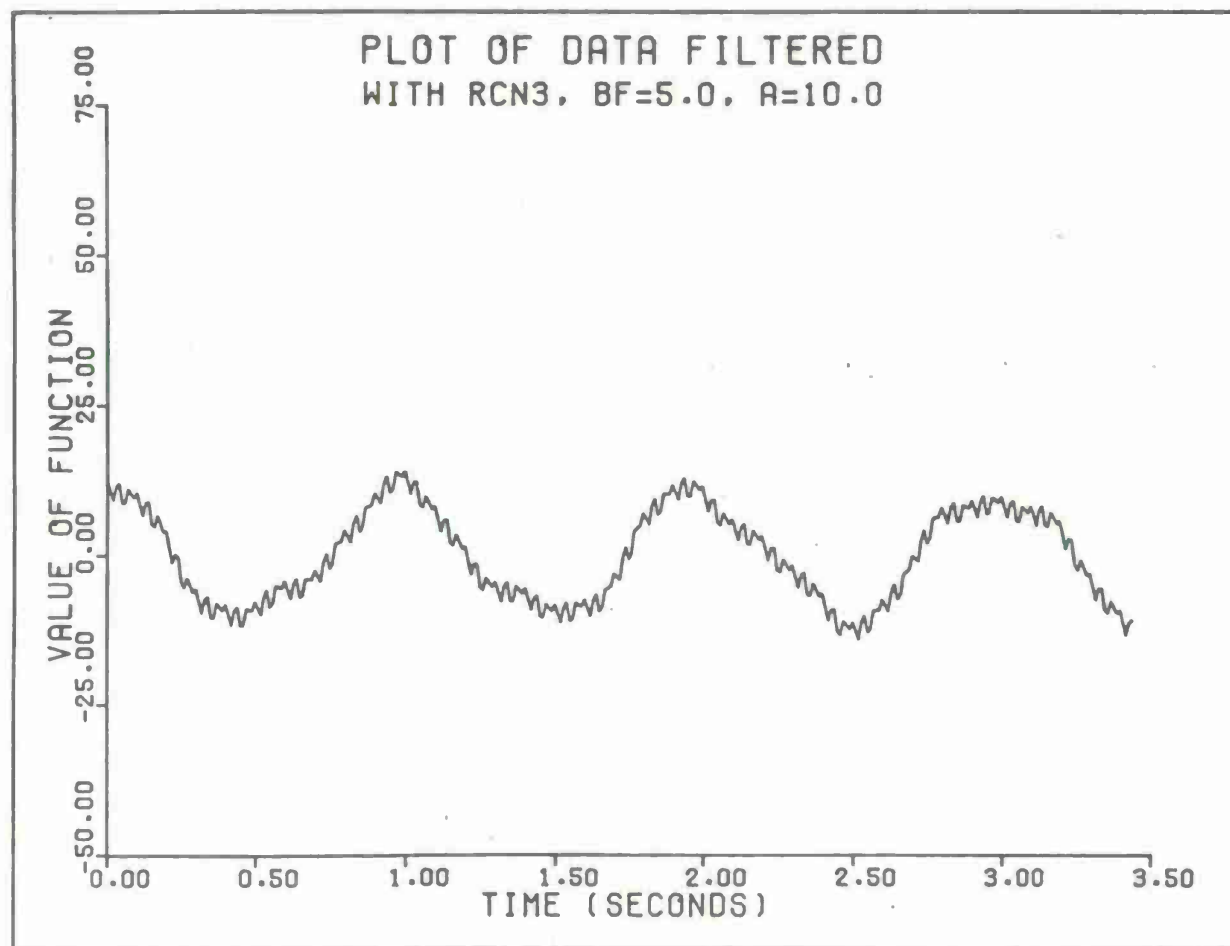


Figure 9 Plot of Data Filtered with RCN3, BF=5.0, A=10.0

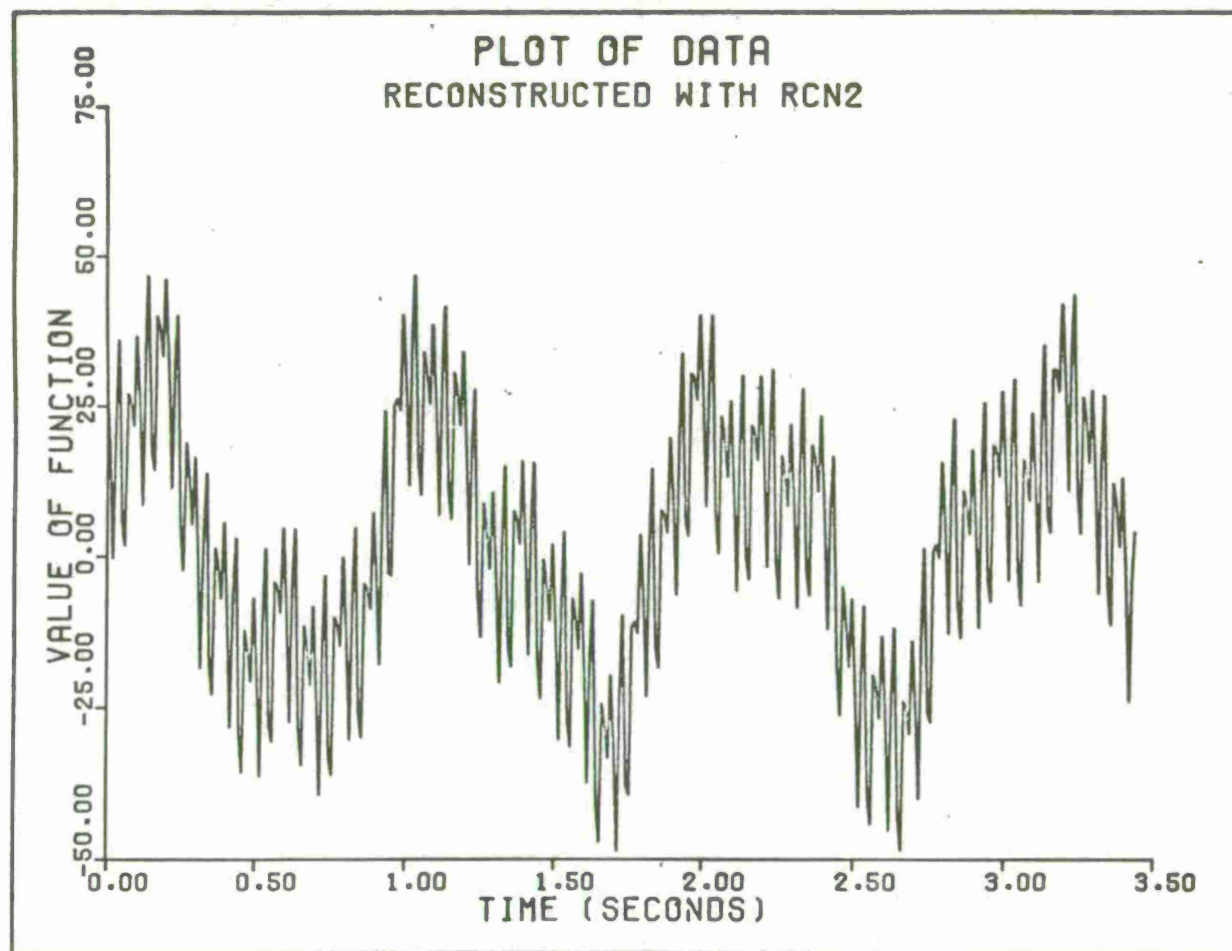


Figure 10 Plot of Data Reconstructed with RCN2

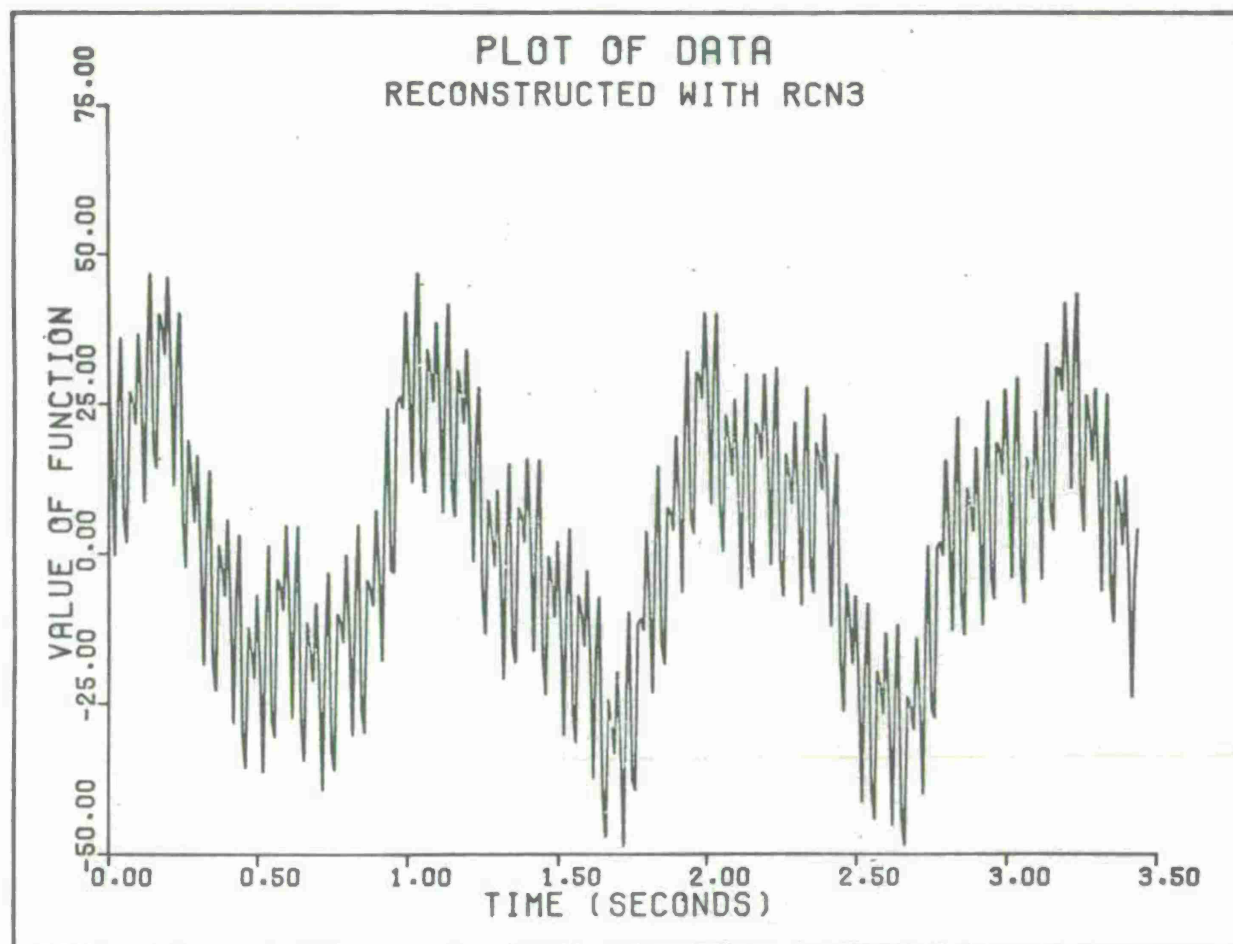


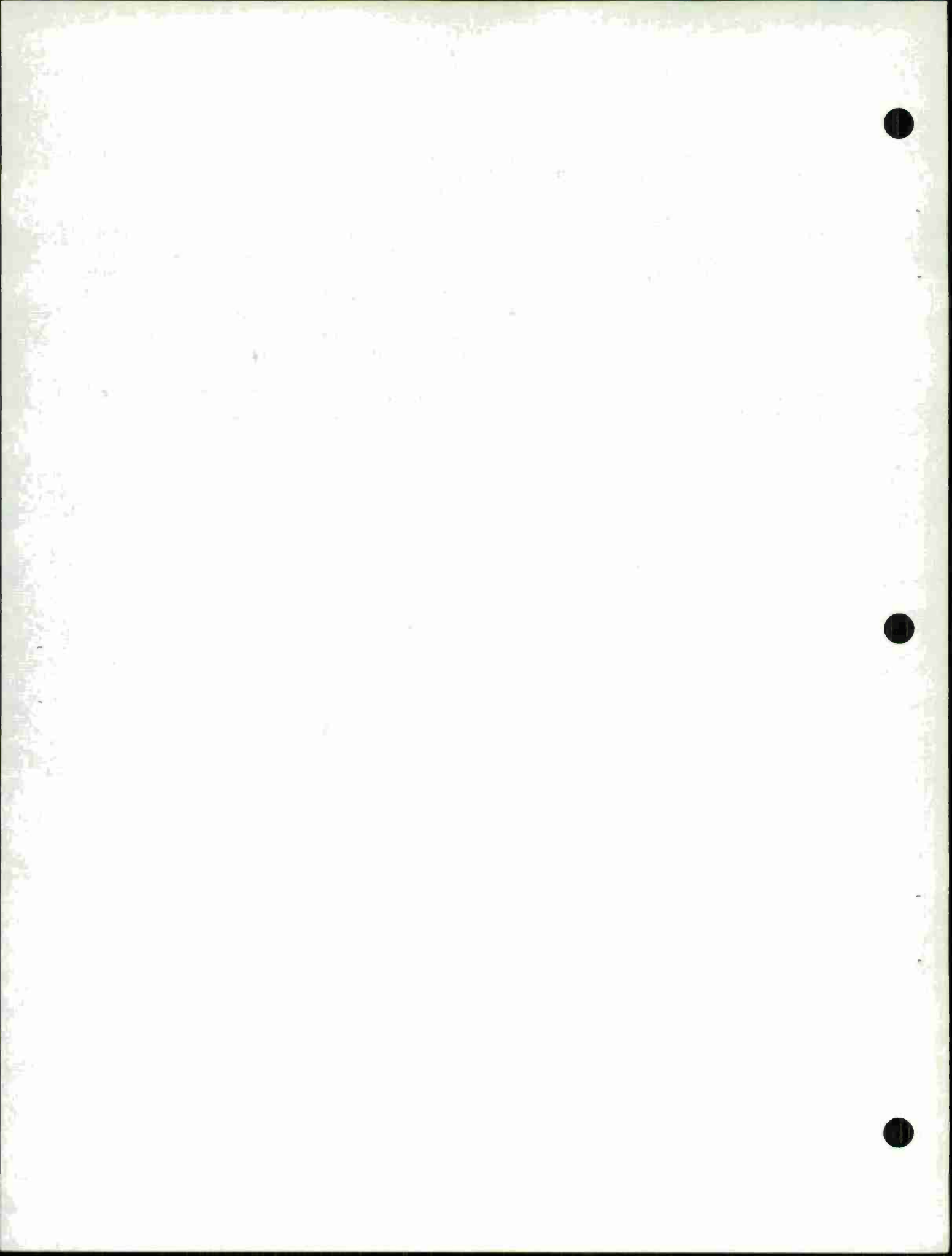
Figure 11 Plot of Data Reconstructed with RCN3



## 6.0 OTHER NETWORKS

Subroutines for the digital simulation of other RC networks could be written by a relatively simple modification of RCN1, RCN2, or RCN3. The procedure is to obtain the frequency transfer function; rationalize the denominator; and, define the transfer function by its real and imaginary parts (AMPR and AMPI). Then do the same for the inverse transfer function and simply replace the 4 statements where AMPR and AMPI are defined in one of the subroutines RCN1, RCN2, or RCN3. Note that complex numbers in these subroutines are treated as two real numbers so they can be used on computer systems which do not have a complex number capability. Note, also, that in each of the subroutines, use is made of the fact that only the imaginary part of the transfer function changes sign for negative frequencies. Thus, the Fourier coefficients corresponding to negative frequencies in the modified FFT are the complex conjugates of the coefficients corresponding to positive frequencies.





## 7.0 PRACTICAL APPLICATIONS

Mathematical models of a wide range of weapon systems have been developed. These systems range from small arms to complete tanks and helicopter gun ships. These models are useful in sensitivity studies and in the evaluation of competing subsystems. It is customary to first validate a math model by comparing the output of the model with experimental data from laboratory or field tests and refining the model until satisfactory agreement is obtained.

As the system becomes larger and more complicated the accuracy of the modeling may become more approximate because it is not practical or necessary to include every detail of a large system in a model. For example, a model of a tank (HITPRO) does not include any provision to account for vibration from the engine and drive train, or structural vibration of the hull and turret which are modeled as rigid bodies. This is not considered to be a limitation of the model because, for its intended purpose, it is not necessary to predict high frequency response. However, high frequency vibration from these sources does show up in the experimental data. Also, experimental data may contain noise caused by the instrumentation, signal processing, recording, etc. Therefore, when comparing the experimental data with the model output it is reasonable to filter out these higher frequencies. Figure 12 shows the output from an accelerometer mounted in the turret of a MICV (Mechanized Infantry Combat Vehicle). The spectral plot of this data, Figure 13, shows a considerable amount of frequency content in the range above 20 Hertz which is obviously structural vibration and noise. Figure 14 shows the result of filtering this data with PLPASS using a cut off frequency of 8 Hertz so the data could be compared with the model output.

Figure 15 shows the vertical gun pointing error of an M60A2 tank with a stabilized gun as predicted by a mathematical model of the system (HITPRO). It was desirable to obtain an RMS value for the pointing error as a measure of the performance of the gun stabilization system. This value was to be compared with a similar value for a competing system. There is reason to believe that the upward drift shown in Figure 15 was caused by some inaccuracy in the model of the system and the inability of the gunner to respond to small errors - not the stabilization system. Therefore, to obtain a meaningful RMS value of pointing error as a measure of the performance of the stabilization system, it was desirable to remove this drift before computing the RMS value. If this drift was linear, one could simply subtract an appropriate ramp function from the data. The drift, however, appears to be nonlinear and, therefore, some other method was required. Filtering out the very low frequencies was found to be an effective method of removing drift from this data. The result of filtering the data with the Perfect Band PASS (PBPASS) filter is shown in Figure 16. A low frequency cut off of 0.25 HZ was found to be adequate. A high frequency cut off of 25.0 HZ was used because the actual system being modeled is not capable of responding to frequencies this high.

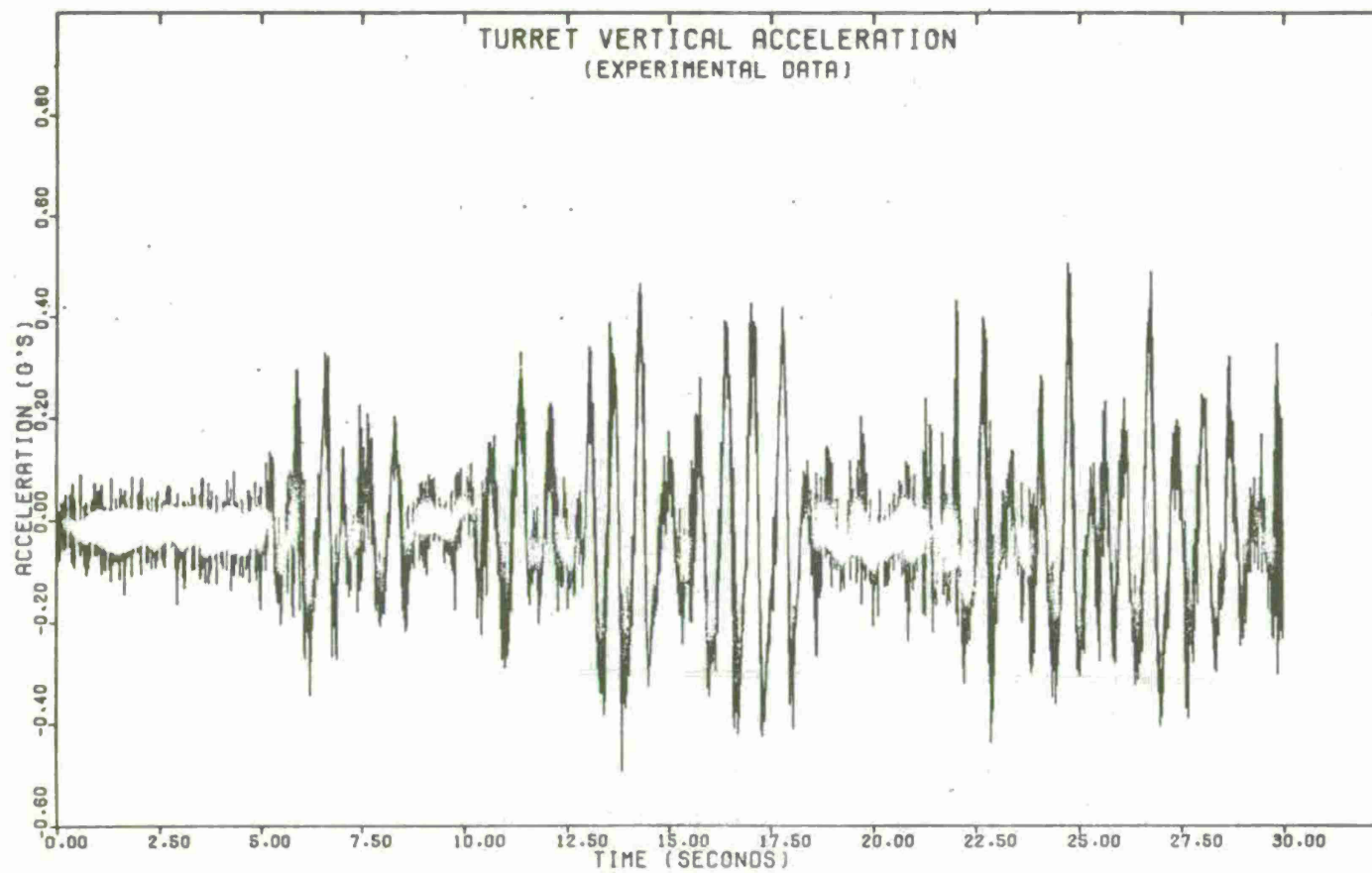


Figure 12 Turret Vertical Acceleration (Experimental Data)

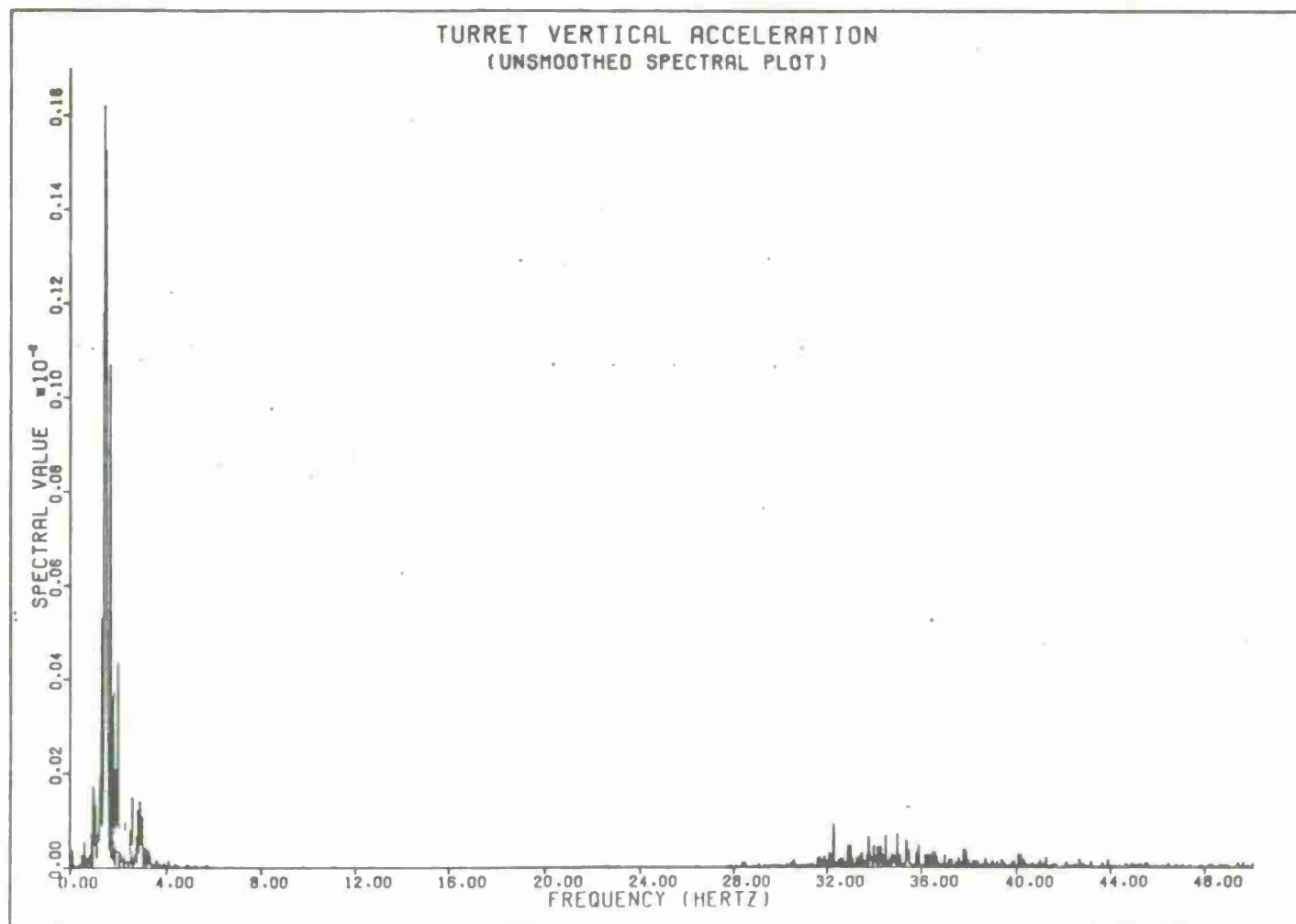


Figure 13 Turret Vertical Acceleration (Unsmoothed Spectral Plot)

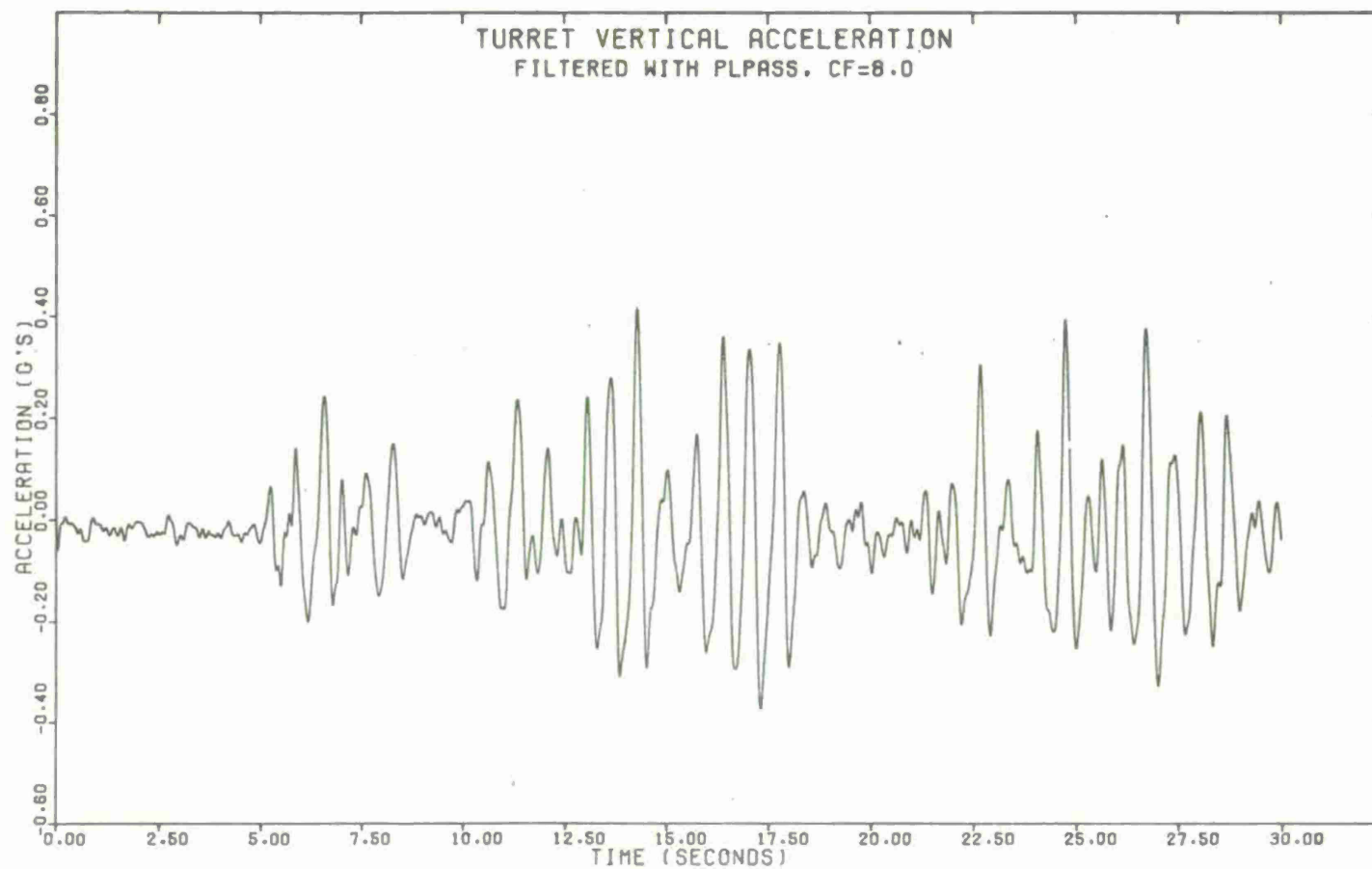


Figure 14 Turret Vertical Acceleration Filtered with PLPASS, CF=8.0

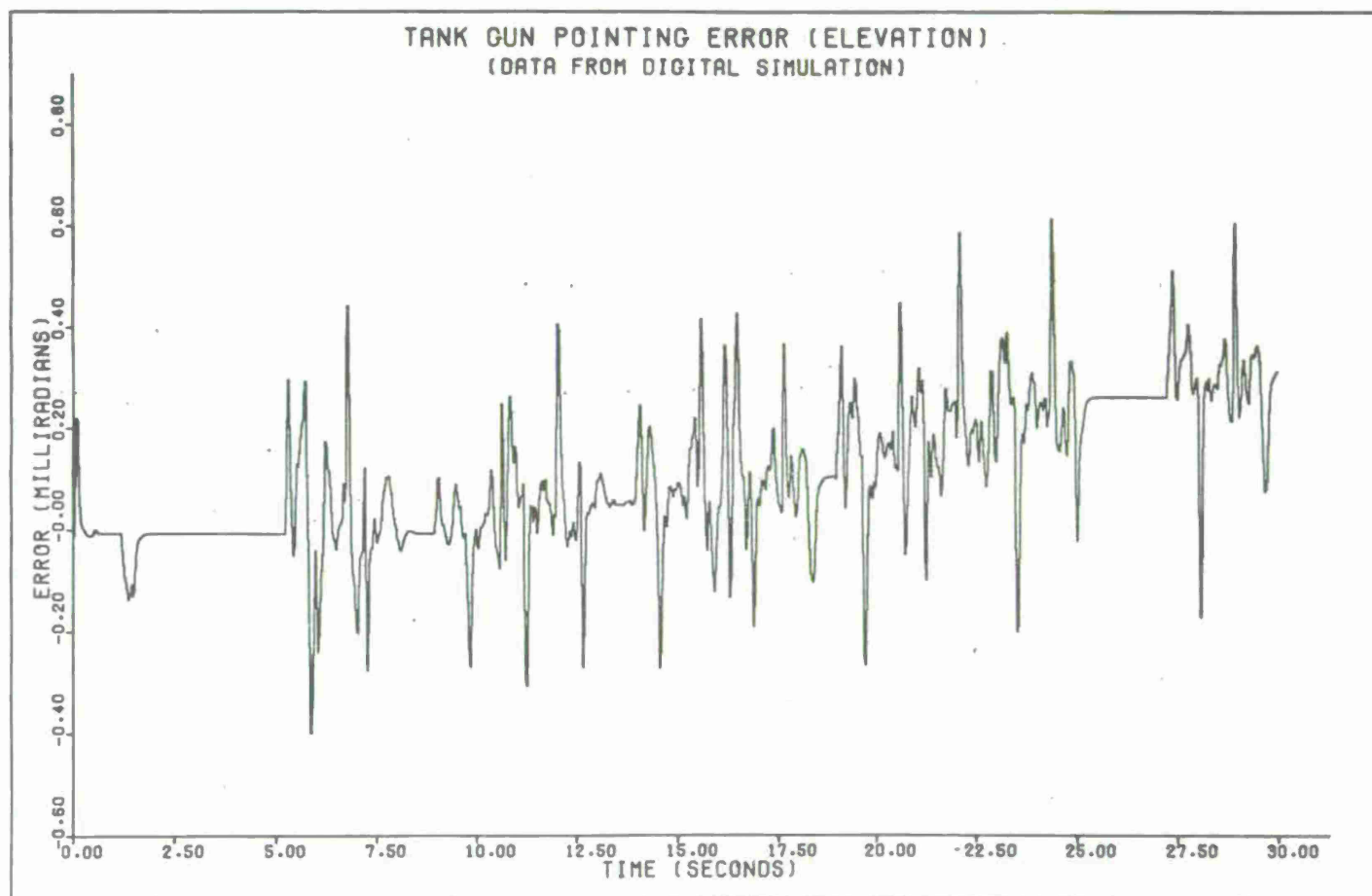


Figure 15 Tank Gun Pointing Error (Elevation) (Data From Digital Simulation)

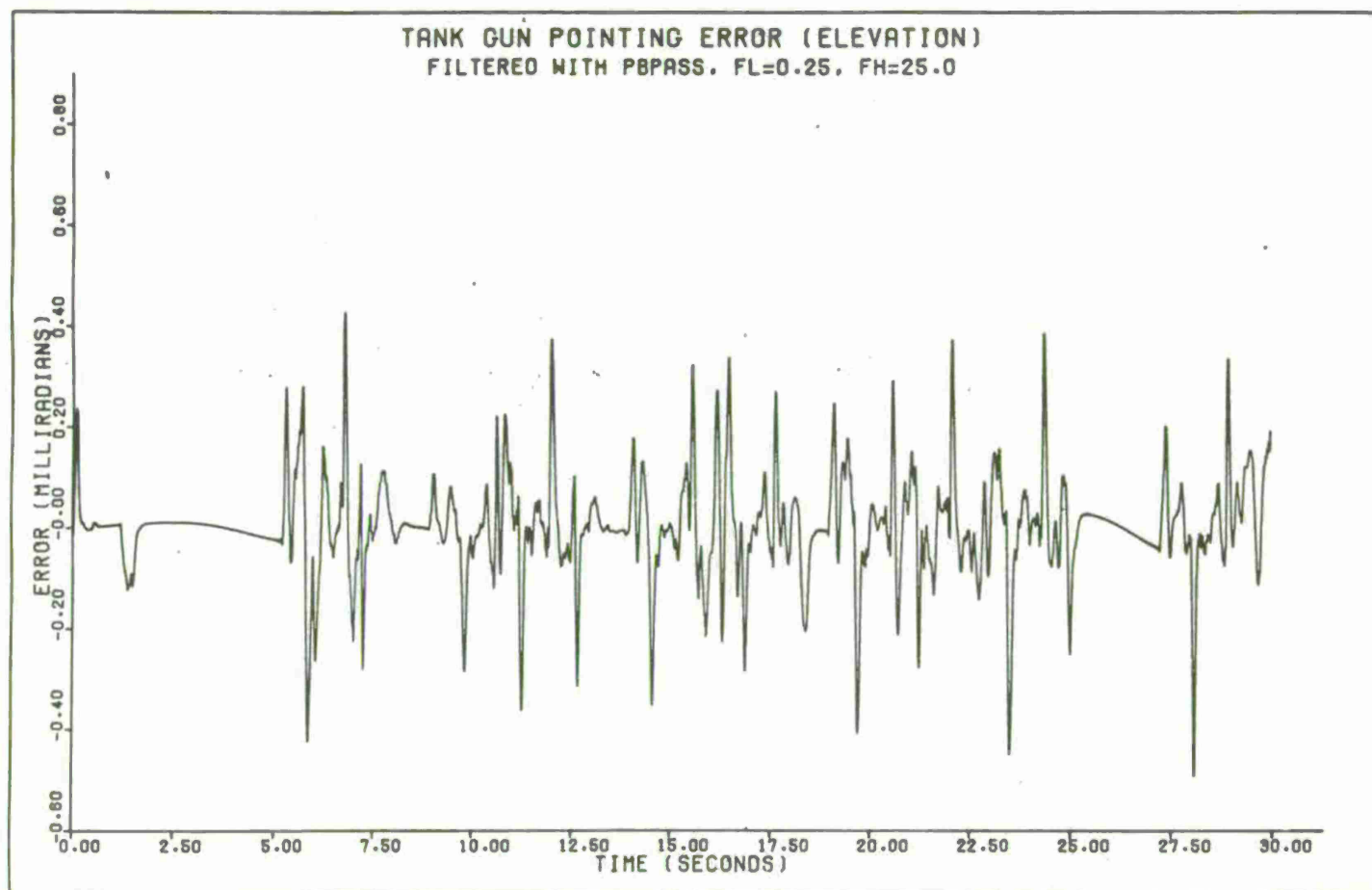


Figure 16 Tank Gun Pointing Error (Elevation) Filtered with PBPASS, FL=0.25, FH=25.0



## 8.0 CONCLUSION

The techniques presented in this paper provide an efficient means of altering the frequency content of discrete data traces in a completely predictable manner. These techniques are applicable in many diverse fields and data processing requirements. In addition to the applications to data from combat vehicles, these techniques have been successfully used to smooth aircraft flight path data, as measured by tracking radar. Smoothing was necessary because this data was used as an input into a mathematical model, and it was physically impossible for an aircraft to follow the flight path indicated by the raw data. These techniques have also been used in the reduction of helicopter flight test data.



APPENDIX A

```

SUBROUTINE FOUR1(DATA,N,ISIGN)
C THE COOLEY-TUKEY FAST FOURIER TRANSFORM IN USASI BASIC FORTRAN.
C NOTE-- IT SHOULD NOT BE NECESSARY TO CHANGE ANY STATEMENT IN THIS
C PROGRAM SO LONG AS THE FORTRAN COMPILER USED STORES REAL AND
C IMAGINARY PARTS ADJACENTLY IN STORAGE.
C  $TRANSFORM(K) = \sum (DATA(J) \cdot \exp(i \cdot ISIGN \cdot 2 \cdot \pi \cdot \sqrt{-1} \cdot (J-1) \cdot (K-1) / N))$ , SUMMED OVER ALL J AND K FROM 1 TO N. DATA IS A ONE-
C DIMENSIONAL COMPLEX ARRAY (I.E., THE REAL AND IMAGINARY PARTS ARE
C ADJACENT IN STORAGE, SUCH AS FORTRAN IV PLACES THEM) WHOSE LENGTH
C  $N = 2^m \cdot K$ ,  $K \neq 0$  (IF NECESSARY, APPEND ZEROES TO THE DATA). ISIGN
C IS +1 OR -1. IF A -1 TRANSFORM IS FOLLOWED BY A +1 ONE (OR VICE
C VERSA) THE ORIGINAL DATA REAPPEAR, MULTIPLIED BY N. TRANSFORM
C VALUES ARE RETURNED IN ARRAY DATA, REPLACING THE INPUT. THE TIME
C IS PROPORTIONAL TO  $N \cdot \log_2(N)$ , RATHER THAN THE NAIVE  $N^2$ .
C ACCURACY IS ALSO GREATLY IMPROVED, THE RMS RELATIVE ERROR BEING
C BOUNDED BY  $6 \cdot \sqrt{2} \cdot \log_2(N) \cdot 2^{-B}$ , WHERE B IS THE NUMBER OF
C BITS IN THE FLOATING POINT FRACTION. WRITTEN BY NORMAN BRENNER OF
C MIT LINCOLN LABORATORY. JULY 1967. THIS IS THE SHORTEST VERSION
C OF THE FFT KNOWN TO THE AUTHOR. FASTER PROGRAMS FOUR2 AND FOURT
C EXIST THAT OPERATE ON ARBITRARILY SIZED MULTIDIMENSIONAL ARRAYS.
C SEE-- IEEE AUDIO TRANSACTIONS (JUNE 1967), SPECIAL ISSUE ON FFT.
DIMENSION DATA(1)
IP0=2
IP3=IP0*N
I3REV=1
DO 60 I3=1,IP3,IP0
IF(I3-I3REV)10,20,20
10 TEMPR=DATA(I3)
TEMPI=DATA(I3+1)
DATA(I3)=DATA(I3REV)
DATA(I3+1)=DATA(I3REV+1)
DATA(I3REV)=TEMPR
DATA(I3REV+1)=TEMPI
20 IP1=IP3/2
30 IF(I3REV-IP1)60,60,40
40 I3REV=I3REV-IP1
IP1=IP1/2
IF(IP1-IP0)60,30,30
60 I3REV=I3REV+IP1
IP1=IP0
60 IF(IP1-IP3)70,100,100
70 IP2=IP1*2
THETA=6.283185307/FLOAT(ISIGN*IP2/IP0)
SINTH=SIN(THETA/2.)
WSTPR=-2.*SINTH*SINTH
WSTPI=SIN(THETA)
WR=1.
WI=0.
DO 90 I1=1,IP1,IP0
DO 80 I3=I1,IP3,IP2
I2A=I3
I2B=I2A+IP1
TEMPR=WR*DATA(I2B)-WI*DATA(I2B+1)
TEMPI=WR*DATA(I2B+1)+WI*DATA(I2B)
DATA(I2B)=DATA(I2A)-TEMPR
DATA(I2B+1)=DATA(I2A+1)-TEMPI
DATA(I2A)=DATA(I2A)+TEMPR
80 DATA(I2A+1)=DATA(I2A+1)+TEMPI
TEMPR=WR
WR=WR+WSTPR-WI*WSTPI+WR
90 WI=WI+WSTPR+TEMPR+WSTPI+WI
IP1=IP2
DO TO 60
100 RETURN
END

```

SUBROUTINE PLPASS(X,N,DT,CF)

C  
C LOW-PASS DIGITAL FILTER--USES THE COOLEY-TUKEY FAST FOURIER TRANSFORM  
C (FOUR1). X IS THE DATA TRACE TO BE FILTERED AND IS ASSUMED TO OCCUPY THE  
C FIRST HALF OF THE X ARRAY. IT IS NOT NECESSARY TO INITIALIZE THE SECOND  
C HALF OF THE X ARRAY OR TO PUT IT INTO COMPLEX FORM AS REQUIRED BY FOUR1.  
C X MUST BE DIMENSIONED 2=N IN THE CALLING PROGRAM WHERE N IS THE NUMBER OF  
C DATA POINTS. NOTE N MUST EQUAL 2=K WHERE K IS AN INTEOER>ZERO. (APPEND  
C ZEROS TO THE DATA IF NECESSARY.)  
C THE FILTERED DATA IS RETURNED TO THE FIRST HALF OF THE X ARRAY.  
C DT IS THE SAMPLING RATE IN SECONDS.  
C CF IS THE CUTOFF POINT (HZ). OTHER ROUTINES EXIST (PHPASS AND PBPASS)  
C WHICH ARE HIGH-PASS AND BAND-PASS FILTERS RESPECTIVELY.

C  
C DIMENSION X(1)  
C NO=2=N  
C FN=FLOAT(N)  
C DO 1 I=1,N  
C X(NO-2=I+1)=X(N-I+1)  
1 X(NO-2=I+2)=0.0  
C CALL FOUR1(X,N,1)  
C IF=CF\*FN\*DT  
C I1=IF+2  
C I2=N-IF  
C DO 2 I=I1,I2  
C X(2=I-1)=0.0  
2 X(2=I)=0.0  
C CALL FOUR1(X,N,-1)  
C DO 3 I=1,N  
3 X(I)=X(2=I-1)/FN  
C RETURN  
C END

SUBROUTINE PHPASS(X,N,DT,CF)

C  
C HIGH-PASS DIGITAL FILTER--USES THE COOLEY-TUKEY FAST FOURIER TRANSFORM  
C (FOUR1). X IS THE DATA TRACE TO BE FILTERED AND IS ASSUMED TO OCCUPY THE  
C FIRST HALF OF THE X ARRAY. IT IS NOT NECESSARY TO INITIALIZE THE SECOND  
C HALF OF THE X ARRAY OR TO PUT IT INTO COMPLEX FORM AS REQUIRED BY FOUR1.  
C X MUST BE DIMENSIONED 2\*N IN THE CALLING PROGRAM WHERE N IS THE NUMBER OF  
C DATA POINTS. NOTE N MUST EQUAL 2\*\*K WHERE K IS AN INTEGER > ZERO. (APPEND  
C ZEROS TO THE DATA IF NECESSARY.)  
C THE FILTERED DATA IS RETURNED TO THE FIRST HALF OF THE X ARRAY.  
C DT IS THE SAMPLING RATE IN SECONDS.  
C CF IS THE CUTOFF POINT (HZ). OTHER ROUTINES EXIST (PLPASS AND PBPASS)  
C WHICH ARE LOW-PASS AND BAND-PASS FILTERS RESPECTIVELY.  
C

DIMENSION X(1)  
NO=2\*N  
FN=FLOAT(N)  
DO 1 I=1,N  
X(NO-2\*I+1)=X(N-I+1)  
1 X(NO-2\*I+2)=0.0  
CALL FOUR1(X,N,1)  
IF=CF\*FN\*DT  
I2=N-IF+2  
DO 2 I=1,IF  
X(2\*I-1)=0.0  
2 X(2\*I)=0.0  
DO 3 I=I2,N  
X(2\*I-1)=0.0  
3 X(2\*I)=0.0  
CALL FOUR1(X,N,-1)  
DO 4 I=1,N  
4 X(I)=X(2\*I-1)/FN  
RETURN  
END

SUBROUTINE PBPASS(X,N,OT,FL,FH)

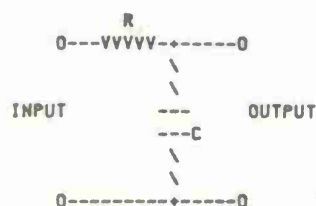
C  
C BANO-PASS DIGITAL FILTER--USES THE COOLEY-TUKEY FAST FOURIER TRANSFORM  
C (FOUR1). X IS THE DATA TRACE TO BE FILTERED AND IS ASSUMED TO OCCUPY THE  
C FIRST HALF OF THE X ARRAY. IT IS NOT NECESSARY TO INITIALIZE THE SECOND  
C HALF OF THE X ARRAY OR TO PUT IT INTO COMPLEX FORM AS REQUIRED BY FOUR1.  
C X MUST BE DIMENSIONED 2\*N IN THE CALLING PROGRAM WHERE N IS THE NUMBER OF  
C DATA POINTS. NOTE N MUST EQUAL 2\*\*K WHERE K IS AN INTEGER>ZERO. (APPEND  
C ZEROS TO THE DATA IF NECESSARY.)  
C THE FILTERED DATA IS RETURNED TO THE FIRST HALF OF THE X ARRAY.  
C OT IS THE SAMPLING RATE IN SECONDS.  
C FL IS THE LOW FREQUENCY CUTOFF POINT (HZ), AND FH IS THE HIGH FREQUENCY  
C CUTOFF POINT. OTHER ROUTINES EXIST (PLPASS AND PHPASS) WHICH ARE  
C LOW-PASS AND HIGH-PASS FILTERS RESPECTIVELY.

C  
C DIMENSION X(1)  
C NO=2\*N  
C FN=FLOAT(N)  
C DO 1 I=1,N  
C X(NO-2\*I+1)=X(N-I+1)  
1 X(NO-2\*I+2)=0.0  
C CALL FOUR1(X,N,1)  
C IF=FL\*FN\*OT  
C I2=N-IF+2  
C DO 2 I=1,IF  
C X(2\*I-1)=0.0  
2 X(2\*I)=0.0  
C DO 3 I=I2,N  
C X(2\*I-1)=0.0  
3 X(2\*I)=0.0  
C IF=FH\*FN\*OT  
C I1=IF+2  
C I2=N-IF  
C DO 4 I=I1,I2  
C X(2\*I-1)=0.0  
4 X(2\*I)=0.0  
C CALL FOUR1(X,N,-1)  
C DO 5 I=1,N  
5 X(I)=X(2\*I-1)/FN  
C RETURN  
C END



SUBROUTINE RCN1(X,N,OT,BF,CF,ISION)

A SIMPLE LOW-PASS RC NETWORK AS SHOWN BELOW:



FREQUENCY TRANSFER  
FUNCTION

$$F(JW) = (1/(1+JW/BF))^{\text{ISION}}$$

WHERE:

$$BF = 1/(R \cdot C)$$

$$J = \text{SQRT}(-1)$$

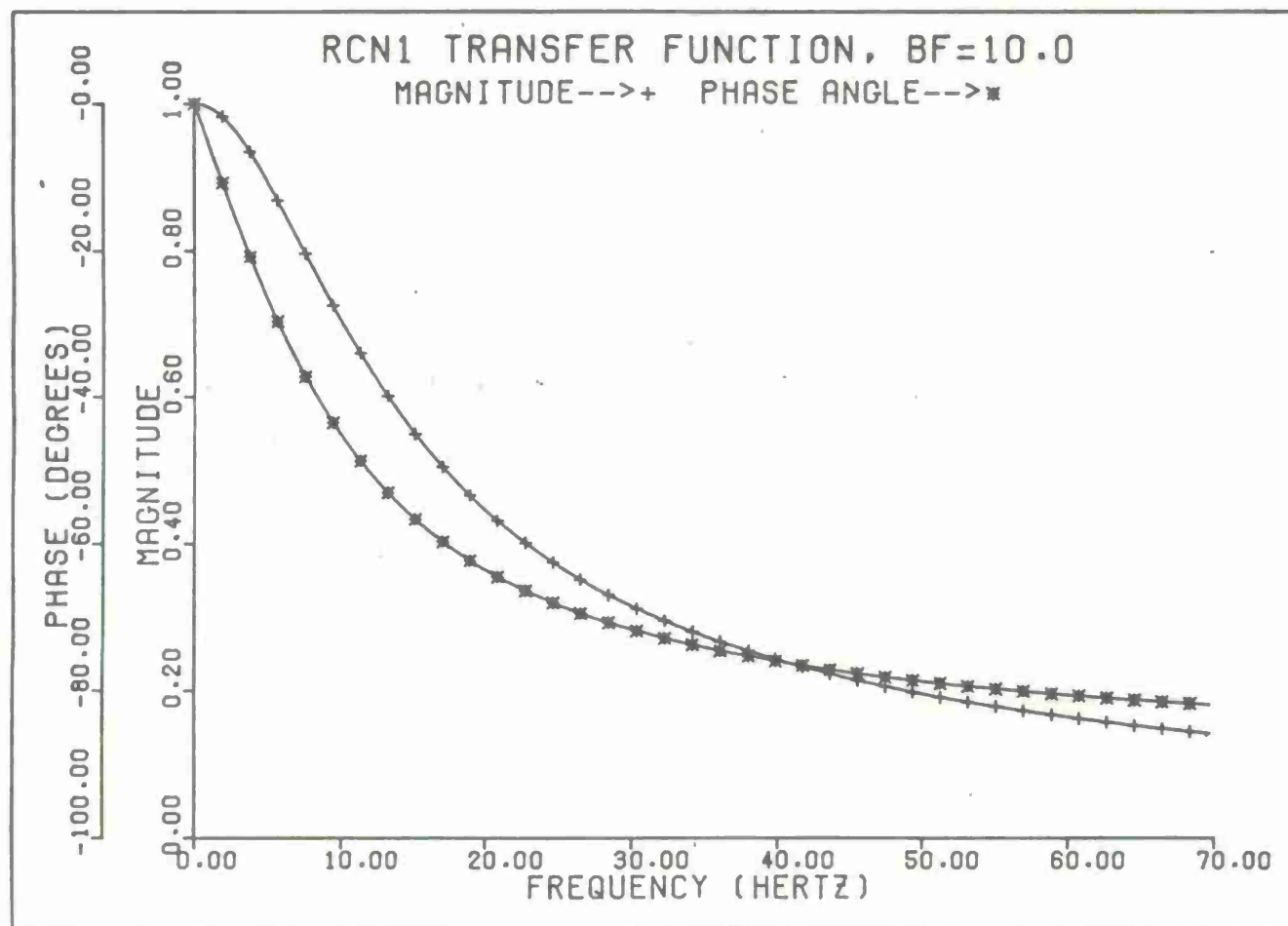
$$W = \text{FREQUENCY (HZ)}$$

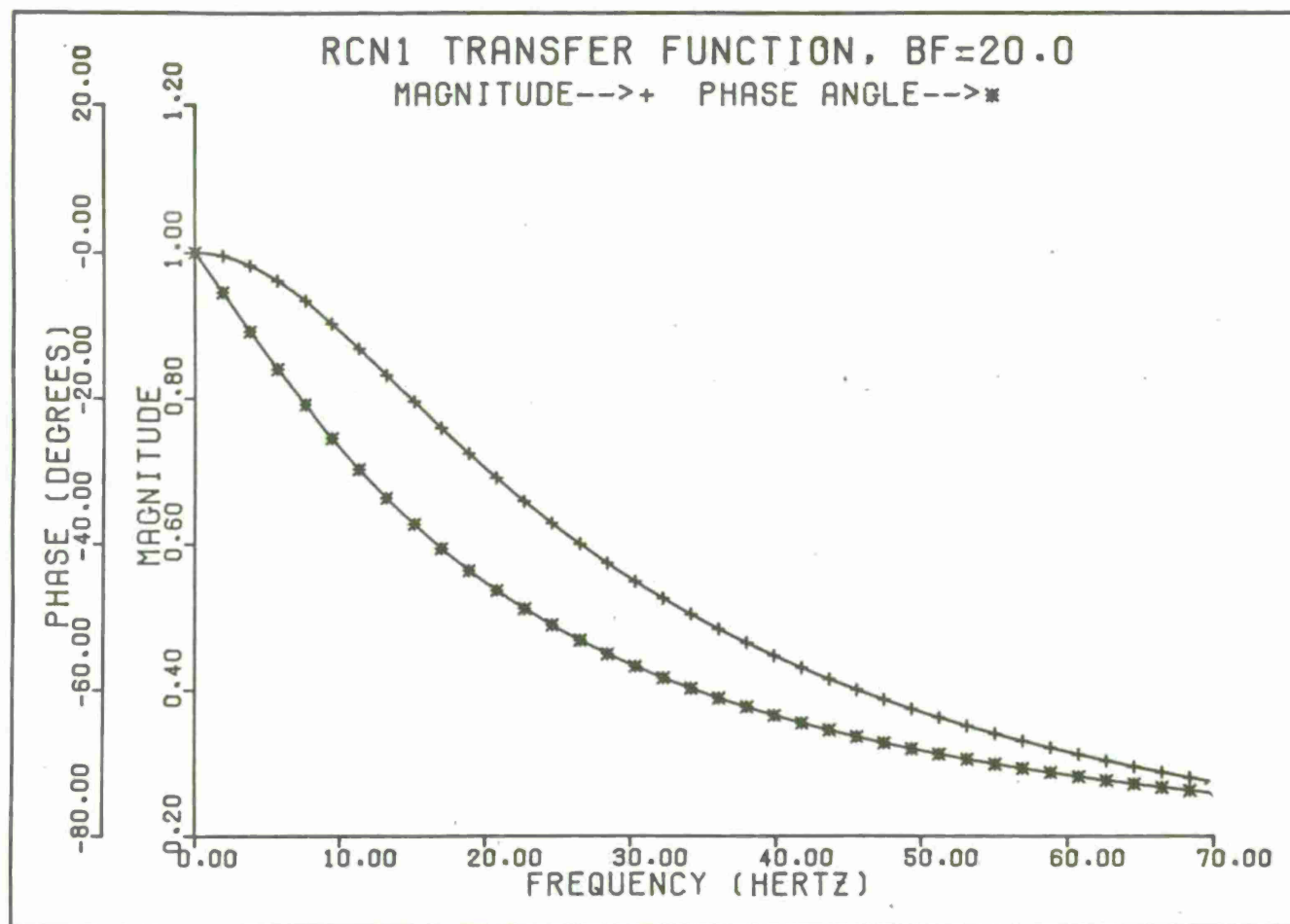
ISION IS +1 OR -1; IF ISION IS +1 THE X VECTOR IS AN ARRAY CONTAINING THE INPUT DATA, AND THE OUTPUT IS TO BE DETERMINED. IF ISION IS -1 THE X VECTOR IS THE OUTPUT FROM THE NETWORK, AND THE INPUT IS TO BE DETERMINED. N IS THE NUMBER OF DATA POINTS AND MUST BE EQUAL TO 2\*\*K WHERE K IS AN INTEGER GREATER THAN 0 (APPEND ZEROS TO THE DATA IF NECESSARY.). X MUST BE DIMENSIONED AT LEAST 2\*N IN THE MAIN PROGRAM. THIS ROUTINE USES THE COOLEY-TUKEY FAST FOURIER TRANSFORM (FOUR1); HOWEVER, IT IS NOT NECESSARY TO PUT THE DATA INTO COMPLEX FORM AS REQUIRED BY FOUR1. THE INPUT DATA IS ASSUMED TO BE IN THE LEFT HALF OF THE X ARRAY, AND THE TRANSFORMED DATA IS RETURNED TO THE LEFT HALF OF THE X ARRAY, REPLACING THE INPUT. IT IS NOT NECESSARY TO DO ANYTHING WITH THE RIGHT HALF OF THE X ARRAY. OT IS THE SAMPLING RATE OF THE DATA (SEC). BF IS THE BREAK FREQUENCY (HZ). CF IS AN OPTIONAL SHARP CUT OFF FREQUENCY (HZ).

```

DIMENSION X(1)
NO=2*N
DO 1 I=1,N
X(NO-2*I+1)=X(N-I+1)
1 X(NO-2*I+2)=0.0
CALL FOUR1(X,N,1)
FM=FLOAT(N)
I2=N/2+1
I=2
2 AMPR=0.0
  AMPI=0.0
  W=FLOAT(I-1)/(FM*OT)
  IF(W*OT*CF) GO TO 6
  IF(ISION) 4,4,3
3 AMPR=1.0/(1.0+(W/BF)**2)
  AMPI=(-(W/BF)/(1.0+(W/BF)**2))
  GO TO 6
4 AMPR=1.0
  AMPI=W/BF
5 TEMP1=X(2*I)
  X(2*I)=X(2*I)*AMPR+X(2*I-1)*AMPI
  X(2*I-1)=X(2*I-1)*AMPR-TEMP1*AMPI
  IF(I.EQ.I2) GO TO 6
  X(NO-2*I+4)=-X(2*I)
  X(NO-2*I+3)=X(2*I-1)
  I=I+1
  GO TO 2
6 CALL FOUR1(X,N,-1)
DO 7 I=1,N
7 X(I)=X(2*I-1)/FM
RETURN
END

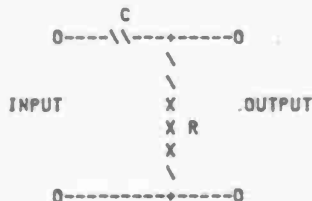
```





SUBROUTINE RCN2(X,N,DT,BF,CF,ISION)

A SIMPLE HIGH-PASS RC NETWORK AS SHOWN BELOW:



FREQUENCY TRANSFER  
FUNCTION

$$F(JW) = (JW / (BF + JW))^{ISION}$$

WHERE:

$$BF = 1/(RC)$$

$$J = \text{SQRT}(-1)$$

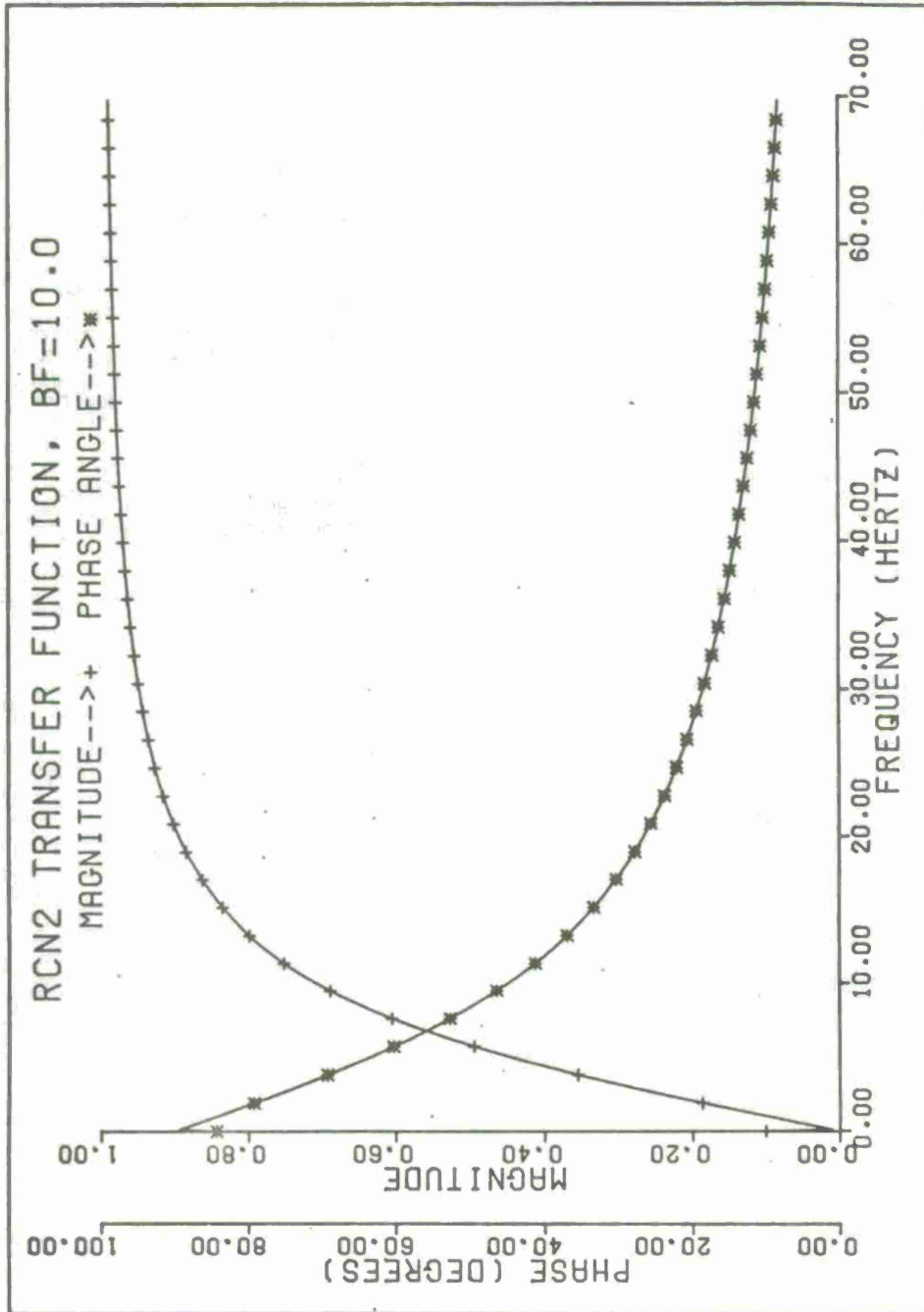
$$W = \text{FREQUENCY (HZ)}$$

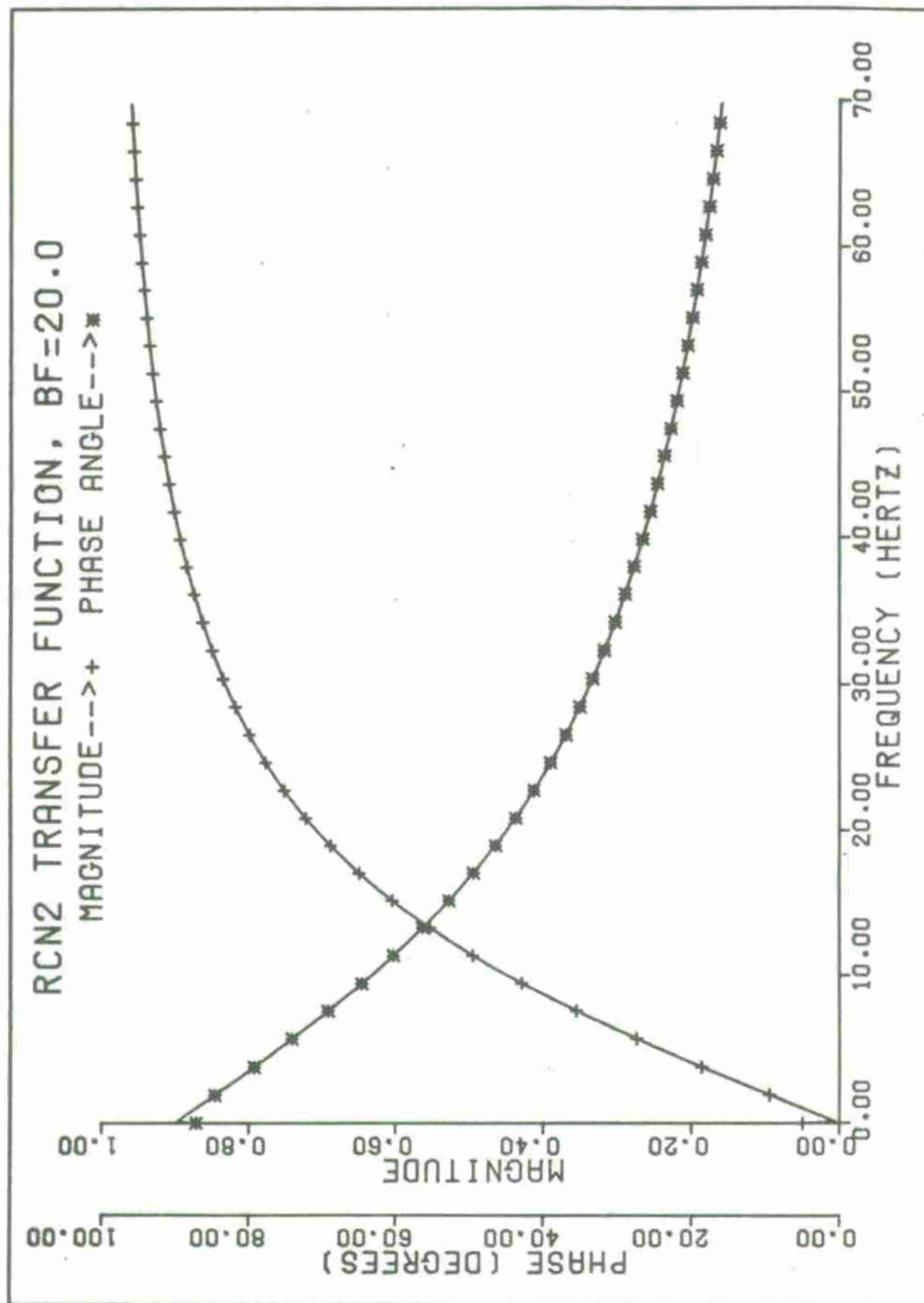
ISION IS +1 OR -1: IF ISION IS +1 THE X VECTOR IS AN ARRAY CONTAINING THE INPUT DATA, AND THE OUTPUT IS TO BE DETERMINED. IF ISION IS -1 THE X VECTOR IS THE OUTPUT FROM THE NETWORK, AND THE INPUT IS TO BE DETERMINED. N IS THE NUMBER OF DATA POINTS AND MUST BE EQUAL TO  $2^m \cdot K$  WHERE K IS AN INTEGER GREATER THAN 0 (APPEND ZEROS TO THE DATA IF NECESSARY.). X MUST BE DIMENSIONED AT LEAST  $2 \cdot N$  IN THE MAIN PROGRAM. THIS ROUTINE USES THE COOLEY-TUKEY FAST FOURIER TRANSFORM (FOUR1); HOWEVER, IT IS NOT NECESSARY TO PUT THE DATA INTO COMPLEX FORM AS REQUIRED BY FOUR1. THE INPUT DATA IS ASSUMED TO BE IN THE LEFT HALF OF THE X ARRAY, AND THE TRANSFORMED DATA IS RETURNED TO THE LEFT HALF OF THE X ARRAY, REPLACING THE INPUT. IT IS NOT NECESSARY TO DO ANYTHING WITH THE RIGHT HALF OF THE X ARRAY. DT IS THE SAMPLING RATE OF THE DATA (SEC). BF IS THE BREAK FREQUENCY (HZ). CF IS AN OPTIONAL SHARP CUT OFF FREQUENCY (HZ).

```

DIMENSION X(1)
NO=2*N
DO I=1,N
  X(NO-2*I+1)=X(N-I+1)
  X(NO-2*I+2)=0.0
CALL FOUR1(X,N,1)
FN=FLOAT(N)
I2=N/2+1
I=2
2  AMPR=0.0
  AMPI=0.0
  W=FLOAT(I-1)/(FN*DT)
  IF(W*DT*CF) GO TO 6
  IF(ISION) 4,3,3
3  AMPR=1.0/(1.0+(BF/W)**2)
  AMPI=1.0/(BF/W+BF)
  GO TO 6
4  AMPR=1.0
  AMPI=-BF/W
5  TEMP1=X(2*I)
  X(2*I)=X(2*I)*AMPR+X(2*I-1)*AMPI
  X(2*I-1)=X(2*I-1)*AMPR-TEMP1*AMPI
  IF(I.EQ.I2) GO TO 6
  X(NO-2*I+4)=-X(2*I)
  X(NO-2*I+3)=X(2*I-1)
  I=I+1
  GO TO 2
6  CALL FOUR1(X,N,-1)
  DO 7 I=1,N
7  X(I)=X(2*I-1)/FN
RETURN
END

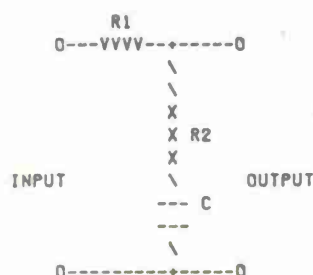
```





SUBROUTINE RCN3(X,N,DT,BF,A,CF,ISION)

AN RC NETWORK AS SHOWN BELOW:



FREQUENCY TRANSFER  
FUNCTION

$$F(JW) = ((1/A) * (BF + JW)) / (BF/A + JW) = ISION$$

WHERE:

$$A = 1 + R1/R2$$

$$BF = 1/(R2 * C)$$

$$J = \text{SQRT}(-1)$$

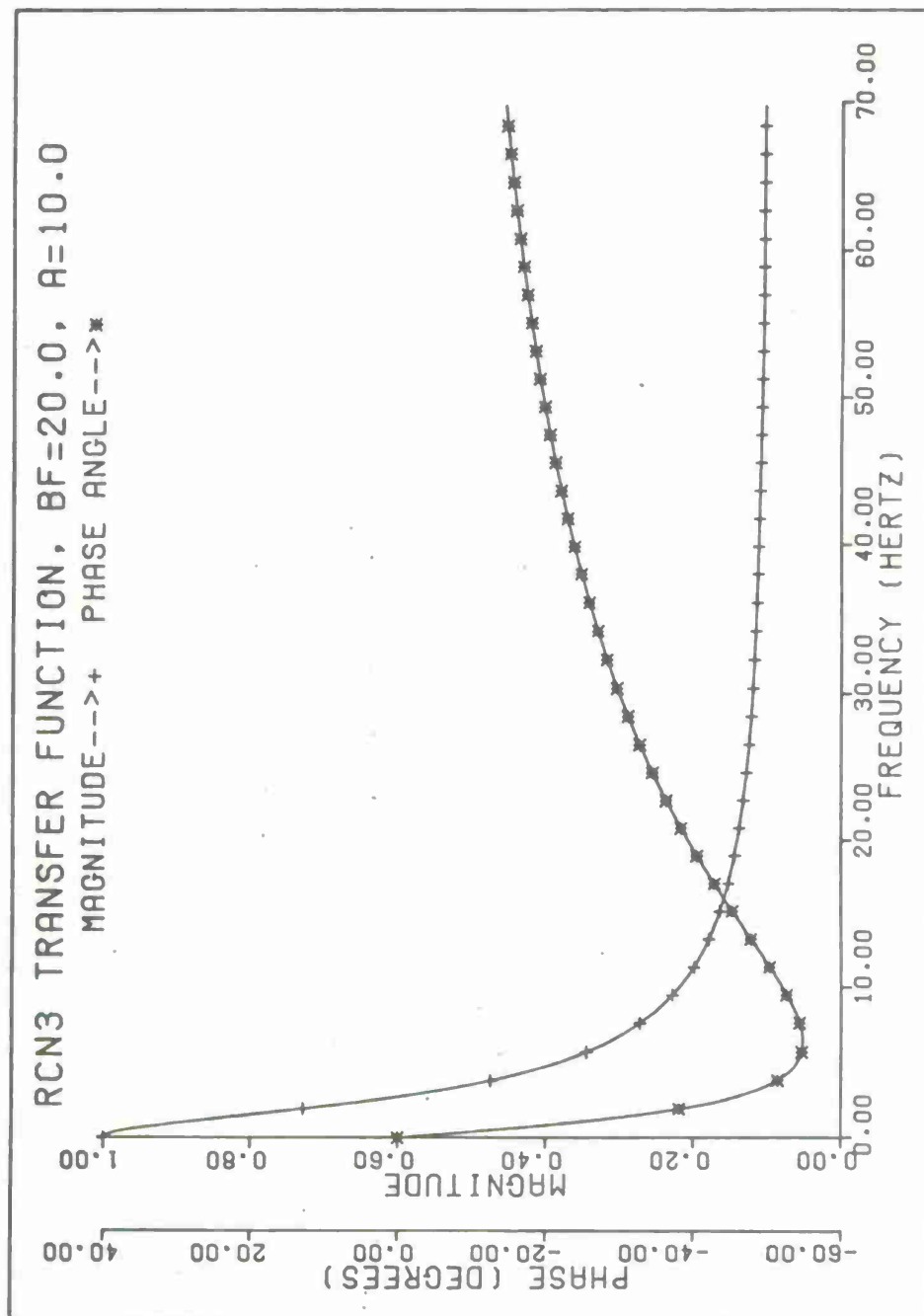
$$W = \text{FREQUENCY (HZ)}$$

ISION IS +1 OR -1; IF ISION IS +1 THE X VECTOR IS AN ARRAY CONTAINING THE INPUT DATA, AND THE OUTPUT IS TO BE DETERMINED. IF ISION IS -1 THE X VECTOR IS THE OUTPUT FROM THE NETWORK, AND THE INPUT IS TO BE DETERMINED. N IS THE NUMBER OF DATA POINTS AND MUST BE EQUAL TO 2\*\*K WHERE K IS AN INTEGER GREATER THAN 0 (APPEND ZEROS TO THE DATA IF NECESSARY.). X MUST BE DIMENSIONED AT LEAST 2\*N IN THE MAIN PROGRAM. THIS ROUTINE USES THE COOLEY-TUKEY FAST FOURIER TRANSFORM (FOUR1); HOWEVER, IT IS NOT NECESSARY TO PUT THE DATA INTO COMPLEX FORM AS REQUIRED BY FOUR1. THE INPUT DATA IS ASSUMED TO BE IN THE LEFT HALF OF THE X ARRAY, AND THE TRANSFORMED DATA IS RETURNED TO THE LEFT HALF OF THE X ARRAY, REPLACING THE INPUT. IT IS NOT NECESSARY TO DO ANYTHING WITH THE RIGHT HALF OF THE X ARRAY. DT IS THE SAMPLING RATE OF THE DATA (SEC). CF IS AN OPTIONAL SHARP CUT OFF FREQUENCY (HZ).

```

DIMENSION X(1)
NO=2*N
DO 1 I=1,N
X(NO-2*I+1)=X(N-I+1)
1 X(NO-2*I+2)=0.0
CALL FOUR1(X,N,1)
FN=FLOAT(N)
I2=N/2+1
I=2
2 AMPR=0.0
AMPI=0.0
W=FLOAT(I-1)/(FN*DT)
IF(W*DT*CF) GO TO 5
IF(ISION) 4,3,3
3 AMPR=(1.0+A*(W/BF)**2)/(1.0+(A*W/BF)**2)
AMPI=W*BF*(1.0-A)/(BF**2+(A*W)**2)
GO TO 6
4 AMPR=(1.0+A*(W/BF)**2)/(1.0+(W/BF)**2)
AMPI=(A-1.0)/(BF/W+W/BF)
5 TEMP1=X(2*I)
X(2*I)=X(2*I)*AMPR+X(2*I-1)*AMPI
X(2*I-1)=X(2*I-1)*AMPR-TEMP1*AMPI
IF(I.EQ.I2) GO TO 6
X(NO-2*I+4)=-X(2*I)
X(NO-2*I+3)=X(2*I-1)
I=I+1
GO TO 2
6 CALL FOUR1(X,N,-1)
DO 7 I=1,N
7 X(I)=X(2*I-1)/FN
RETURN
END
  
```





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Investigation of Digital Filtering Techniques  
For the Analysis of Experimental Data

Prepared By: Larry D. Wells

Technical Report R-TR-75-024

Pages: Incl. Plates

This paper presents techniques for processing digitized data to remove noise and artifacts and to simulate the effect of simple EC work on the corresponding analog data. The development of these techniques provides a means of isolating a signal from background noise, a means of removing low frequency drift from a signal, and a means of selectively reversing previous filtering. These techniques are valuable to the refinement and validation of digital simulations. The method presented in this paper uses the Fast Fourier Transform. The methodology has been applied to remove noise and drift from experimental and mathematical model data relative to

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Investigation of Digital Filtering Techniques  
For the Analysis of Experimental Data

Prepared By: Lanny D. Wells

Technical Report 8-TR-75-024

Beane, Earl Patrick

This paper presents techniques of processing digital data to remove the effects of the noise of the analog-to-digital converter (ADC) and the noise of the corresponding analog data. The development of these techniques provides: a means of isolating a signal from background noise, a means of removing low frequency drift from a signal, and a means of selectively reversing previous filtering. These techniques are valuable in the refinement and validation of the final simulations. The method presented to this paper uses the Fast Fourier Transform. The Monte Carlo method applied to remove noise from digital data is a statistical method. The Monte Carlo method is a different statistical and mathematical approach to remove noise. Determining an RMS value of a tank sum pointing error.

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